

Master-Seminar

Complex quantum systems and vortex dynamics: from analysis to numerical simulation

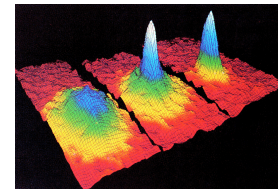
This seminar explores quantum vortices, which are topological defects that occur in the flow of superfluids. For example, helium undergoes a phase transition at temperatures close to absolute zero and becomes a superfluid. As with classical fluids, vortices are observed, but, unlike their classical counterparts, these vortices obey quantum mechanical laws at the macroscopic level. Similar phenomena occur in Bose–Einstein condensates and superconductors. Researchers were awarded the Nobel Prize in Physics for their contributions to these fields in 1978 and 2001. Mathematically, these phenomena are described by the Gross–Pitaevskii problem, a nonlinear Schrödinger equation of the form

$$i \partial_t \psi = -\Delta \psi + \frac{1}{\varepsilon^2} (|\psi|^2 - 1) \psi$$

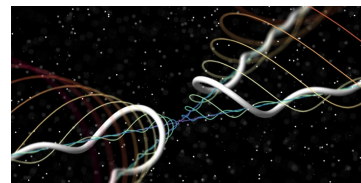
with the wave function ψ , the imaginary unit i , and the interaction parameter $\varepsilon > 0$.

A single vortex constitutes a stationary solution, multiple vortices exhibit a complex dynamics. Understanding these dynamics are crucial for understanding quantum turbulence. At the same time, the Gross–Pitaevskii equation is a universal model with broader applications in superconductivity, nonlinear optics, and related fields.

This seminar explores analytic and numerical methods for analyzing and simulating the properties and dynamics of quantum vortices. Also topological concepts, such as topological degree, will be important. A focus is understanding the distinct behavior of quantized vortices when compared to point vortices in classical fluids, namely, investigating the relevance of quantum effects at the macroscopic level. The literature provided and the talks will be in English. Topics can be adapted to the attendees' mathematical background.



BEC, Nobel prize 2001



Collision of vortices, Enrico Fonda

Prerequisites: Functional analysis and a basic course in PDEs. Knowledge in numerical analysis of ODEs and PDEs is helpful but not required.

Date: The seminar will be held as a block course. That is, we will have talks from May till mid June (from 15:45pm - 18:00pm on Wednesdays).

Preliminary meeting: The preliminary meeting takes place on February 19, 13:15pm in room 3.069. For organizational purposes, we kindly ask you to express your interest in participating in advance by sending an e-mail to Moritz Hauck moritz.hauck@kit.edu.