Variational Solutions of the Dirichlet Problem: Lebesgue's cusp, non-local properties and elliptic operators

Wolfgang Arendt
(University of Ulm, Germany)

Dirichlet's original idea was to find solutions of the Dirichlet Problem by minimizing the energy (the Dirichlet Principle). This is not possible for all boundary data even on the disc in the plane. However, we define a variational solution which always exists and coincides with the classical solution if such a classical solution exists. It satisfies the maximum principle and we also can characterize when it has finite energy. And finally we show that the variational solution coincides with the Perron solution, a generalized solution Perron defined in 1923. An analysis of Lebesgue's cusp, or rather Lebesgue's domain, shows strange behaviour at a singular point (i.e. a point at the boundary where the variational/Perron solution is not continuous with the right value). The first is that there is no way which allows one to decide whether a given boundary function allows a classical solution (no regularity assumption will guarantee this). The second is an extreme non-local behaviour. We will explain these things for the Laplacian, but they are valid for general elliptic operators. The talk is based on joint work with Daniel Daners (Sydney, Tom ter Elst (Auckland) and Manfred Sauter (Ulm).

Multiplication operators on Hardy spaces for Fourier integral operators

Dorothee Frey
(Karlsruhe Institute of Technology, Germany)

We consider Hardy spaces that are invariant under a general class of Fourier integral operators of order zero. These spaces are particularly well suited to the study of wave equations, as they are designed in such a way that that the standard half-wave group is bounded on them. A thorough understanding of multiplications on such spaces is required for perturbation arguments and the study of nonlinear wave equations. In this talk, we discuss the action of multiplication operators on Hardy spaces adapted to structured Lipschitz coefficients and their applications to wave equations with rough coefficients.

Curved Ingham inequalities with applications to the toroidal Schrödinger equation

Bernhard Haak
(Université de Bordeaux, France)

We are interested in upper and lower bounds of L^2-norms of trigonometric series along certain curves when the frequencies are themselves restricted to a power type curve. A typical example of application would be a trigonometric series emerging from the fractional Schrödinger equation

$$\begin{cases} \frac{i}{2\pi}\partial_t u + \frac{1}{(2\pi)^s}|\partial_x|^s u = 0, \quad x \in \mathbb{T}, t \in \mathbb{R}, \\ u(0,x) = u_0 \end{cases}$$

on the torus with initial data $u_0 \in L^2(\mathbb{T})$. The talk relies on joint work with P. Jaming, Y. Wang (both in Bordeaux) and M. Wang (Hunan, China).

Vector-valued holomorphic functions and abstract Fubini-type theorems

Markus Haase
(Kiel University)

Let $f = f(z,t)$ be a function of two parameters, where z is from a certain open subset O of $\mathbb{C}$ and t is from some measure space (Ω, μ). Suppose A is an operator on a Banach space X with a (possibly unbounded) $\mathrm{H}^\infty(O)$-calculus. For certain $x \in X$ and under additional hypotheses on f one can consider expressions of the form

$$\int_\Omega f(A,t)x \, \mu(\mathrm{d}t)$$

and these are central to the Kalton–Weis theory of square function estimates. To avoid technicalities with integrals of vector-valued functions, Haak and Haase in [1,2] proposed an alternative approach to square function estimates, where only expressions of the form

$$\Big(z \mapsto \int_\Omega f(z,t) \, \mu(\mathrm{d}t)\Big)(A)x$$

are employed. This setting also appears to be more general, as, e.g., the function f need not be product measurable.
But how much more general is this alternative approach, really? Pursuing this question led to quite unexpected insights, which I will share with the audience during my talk.
This is joint work with Bernhard Haak (Bordeaux), see [3].

[1] Bernhard H. Haak and Markus Haase: *Square Function Estimates and Functional Calculi*, arXiv1311.0453, 2013.

[2] Bernhard H. Haak and Markus Haase: *The Square Functionasl Calculus*, work in progress.

[3] Bernhard H. Haak and Markus Haase: *Vector-valued holomorphic functions and abstract Fubini-type theorems.* Archiv der Mathematik 123 (2024), 275–290.

Perron-Frobenius in Ordered Banach Algebras

Gerd Herzog
(Karlsruhe Institute of Technology, Germany)

In ordered Banach algebras we give an overview of the property that the spectral radius of some element is contained in the spectrum and present the following recent result: For an element a of a Banach algebra (scaled to spectral radius 1) the spectral radius is contained in the spectrum, if the sequence of powers (a^k) is asymptotically not too far from a normal cone.

Analysis of Liquid Crystal Flows

Matthias Hieber
(TU Darmstadt, Germany)

In this talk we discuss two models describing the dynamics of nematic liquid crystal flow, the Q-tensor model and the Ericksen-Leslie model. We establish local and small data global well-posedness result for these mixed-order systems subject to general stress tensors and anisotropic elasticity.

This is joint work with A. Hussein and M. Wrona as well as with J. Li and M. Wilke.

Stability of an equilibrium state of the 3D MHD equations with arbitrary geometry

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In the 3-dimensional bounded domain Ω with smooth boundary $\partial\Omega$, we consider stability problem of solutions to the MHD equations for $U = (u, B)$ denoting the velocity and magnetic field, respectively. Introducing the harmonic vector field

$$X_{har}(\Omega) = \{h \in C^\infty(\bar{\Omega});\, \mathrm{div}\, h = 0,\, \mathrm{rot}\, h = 0,\, h \cdot \nu|_{\partial\Omega} = 0\}$$

with ν denoting the unit outer normal to $\partial\Omega$, we first show that $U_* = (0, B_*)$ with $B_* \in X_{har}(\Omega)$ gives an equilibrium state. Next, we prove that if B_* is small in $L^1(\Omega)$ and if the initial disturbance U_0 is sufficiently small in $L^3(\Omega)$ with $B_0 - B_*$ perpendicular to $X_{har}(\Omega)$, then U_* is exponentially stable. This is the joint work with Senjo Shimizu(Kyoto Univ.) and Taku Yanagisawa(Nara Women Univ.).

A Leibniz-type magic formula

Sylvie Monniaux

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In this talk, I will show how to estimate terms of the form $\operatorname{curl}(u \times b)$ in L^r with the norms of u, ∇u, b, $\operatorname{div} b$ and $\operatorname{curl} b$. This estimate is very useful when dealing with the magnetohydrodynamical system where the term $\operatorname{curl}(u \times b)$ appears as a non linear coupling in the magnetic equation (the induction equation), where b denotes the magnetic field and u the velocity field. Although this is a 3D question, I will present its n-dimensional version for $n \geq 2$.

A weak type (p_0, a) criterion for operators, and applications

El Maati Ouhabaz
(Université de Bordeaux, France)

Let (X, d, μ) be a space of homogeneous type and Ω an open subset of X. Given a bounded operator $T : L^p(\Omega) \to L^q(\Omega)$ for some $1 \leq p \leq q < \infty$, we give a criterion for T to be of weak type (p_0, a) for p_0 and a such that $\frac{1}{p_0} - \frac{1}{a} = \frac{1}{p} - \frac{1}{q}$. These results are illustrated by several applications including estimates of weak type (p_0, a) for Riesz potentials $L^{-\frac{\alpha}{2}}$ or for Riesz transform type operators $\nabla \Delta^{-\frac{\alpha}{2}}$ as well as $L^p - L^q$ boundedness of spectral multipliers $F(L)$ when the heat kernel of L satisfies a Gaussian upper bound or an off-diagonal bound. We also prove boundedness of these operators from the Hardy space H^1_L associated with L into $L^a(X)$. By duality this gives boundedness from $L^{a'}(X)$ into BMO_L.This talk is based on a joint work with B. Haak (univ. Bordeaux).

Growth and instability of semilinear evolution equations

Roland Schnaubelt
(Karlsruhe Institute of Technology, Germany)

We study semilinear evolution equations with compact and merely continuous nonlinearities. It assumed that for a initial values u_0 in a certain bounded set one has global solutions u which are uniformly bounded on compact time intervals. We assume that the resolvent of the governing strongly continuous semigroup T is unbounded on a line $a + i\mathbf{R}$ for some $a > 0$. For a dense set of u_0 we can then show exponential growth of the norms of $u(t)$ with rate a. There are related results for even residual sets of u_0 involving the essential growth bound of T. Hence a topoplogically small nonlinear perturbation does not change much the growth behavior of solutions. The proof relies on intricate constructions of sets of approximate eigenvectors for the semigroup. The results can be applied to the backward damped semilinear wave equation on smooth manifolds.

The talk is based on a recent joint paper with V. Müller and Y. Tomilov.

Regularity Estimates for Parabolic Evolution Equations and the Absolute Functional Calculus

Lutz Weis, KIT

Abstract: The sum theorem of DaPrato-Grisvard for two sectorial operators A and B is a very useful tool in the regularity theory of evolution equation. It holds if one sectorial operator has an absolute functional calculus (as introduced by Kalton and Kucherenko), without additional assumption on the second sectorial operator or the underlying Banach space (other than the usual restriction on the angles of the sectorial operators A, B).

We present criteria for the absolute functional calculus in terms of the p-absolute functional calculus, perturbation and duality theorems, the connection with real interpolation methods and Littlewood-Paley decompositions. The usefulness of these criteria will be illustrated by various examples of evolution equations, also in non-UMD spaces.