Karlsruhe Institute of Technology

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Nonlinear Partial Differential Equations
WS 25/26

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Orbital Stability of Cnoidal Waves against Localized Perturbations

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Abstract

We investigate the stability and long-term behavior of so-called *cnoidal* waves in the Kortegweg de Vries (KdV) and Nonlinear Schrödinger equation (NLS) against localized perturbations. These periodic solutions correspond to shallow water waves and periodic light pulses in nonlinear optical fibers, respectively. To date, nonlinear stability results for periodic waves in Hamiltonian systems have primarily addressed co-periodic or subharmonic perturbations. Their stability with respect to localized perturbations - a natural setting in many physical applications - remains a longstanding open problem, as such perturbations render the wave neither localized nor periodic, placing its stability analysis outside the scope of the classical orbital stability framework for Hamiltonian systems developed by Grillakis, Shatah, and Strauss. We present an alternative approach that combines variational techniques, leveraging conserved quantities tailored to the perturbation equation, with Duhamel-based methods and a modulational ansatz - tools known from the stability analysis of periodic waves in dissipative systems and reaction diffusion models. Using this approach, we establish orbital stability of cnoidal waves with respect to L^2-localized perturbations.