On the NLS Hierarchy with Nonzero Boundary Condition

Robert Wegner, KIT

Abstract

The NLS and KdV equations are the first infinite dimensional Hamiltonian systems that were proven to be completely integrable. A consequence is that they both have an associated infinite hierarchies of PDEs which generate higher symmetries – the NLS and KdV hierarchies.

We review the literature on well-posedness results for these hierarchies, and present a new result for the NLS hierarchy in the setting of the nonzero boundary condition $\lim_{x \to \pm\infty} q(x) = q_\pm$, where $|q_\pm| = 1$.

Subsequently, we consider the behavior of long-wave solutions to this hierarchy from the perspective of the hydrodynamic variables $w_\pm = \frac{1}{2}\Im[q^{-1}q_x] \pm (|q| - 1)$, and discover that the effective dynamics are given by the KdV hierarchy.