Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

April 10th, 2023, 9:45 - 10:45
Seminar room: SR 3.069

Sobolev Instability for the Cubic NLS on Irrational Tori

Filippo Giuliani, Politecnico di Milano

Abstract

In the last two decades the study of instability in Sobolev spaces for nonlinear Hamiltonian partial differential equations on compact manifolds has drawn lots of attention in the mathematical community. A breakthrough result in this direction is due to Colliander-Keel-Staffilani-Takaoka-Tao (Invent. Math 2010), who showed the existence of solutions to the defocusing cubic NLS on the 2-dimensional square torus with arbitrarily small initial data and arbitrarily large high order Sobolev norms at later times. The mechanism to construct such unstable solutions is based on the study of the resonant dynamics of NLS. Staffilani noticed that the same strategy would not apply for the NLS equation on 2-dimensional irrational tori, where there are less resonant waves interactions. In this talk we discuss how we overcame this problem to prove Sobolev instability for the cubic NLS on irrational tori. Moreover, we present a recent result of this type where we also take into account the presence of smooth convolution potentials.

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Nonlinear Partial Differential Equations
SS 24

April 17th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

On the Bäcklund Transform and the Stability of the Line Soliton of the KP-II Equation on $\mathbb{R}^2$

Lorenzo Pompili, University of Bonn

Abstract

We study the Miura map and the resulting soliton-addition-map of the KP-II equation on $\mathbb{R}^2$, with the ultimate aim of proving the modulational stability of the line soliton in L^2. We classify solutions of the Miura map that are close to a modulated kink, obtaining a large family of stable perturbations of the line soliton in L^2, and prove codimension-1 stability under perturbations in a weighted space by studying the range of the soliton-addition-map. The method is flexible enough to allow the study of the stability of multisoliton as well.

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SS 24

April 24th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Hölder Regularity for Collapses of Point-vortices

Ludovic Godard-Cadillac, Bordeaux INP

Abstract

The point-vortex system is a system of differential equations arising in the context of inviscid planar fluid mecanics which models the dynamics for the centers of whirlpools. These equations are derived from the Euler 2D or the Surface Quasi-Geostrophic equations under the assumption that the vorticity of the fluid is sharply concentrated around some points (and formally replaced by Dirac masses). This system is well-posed as long as there are no collapses of point-vortices (between themselves or with the boundary). It is a natural question to wonder what is the behavior of this dynamical system in the neighborhood (in space or time) of a collapse and describe the blow-up. I will present in this talk my contributions to this problem, which are some results from my PhD thesis and very recent developpements in collaboration with Martin Donatin and Dragos Iftimie.

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SS 24

May 8th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Hidden Asymptotics for the Weak Solutions of the Strongly Stratified Boussinesq System Without Rotation

Frédéric Charve, UPEC

Abstract

It is known that when the Froude number goes to zero, the solutions of the strongly stratified Boussinesq system tend towards those of a 3D-Navier-Stokes-type system (but with only two components). Surprisingly, this limit system does not depend on the thermal diffusivity $\nu' > 0$. In this talk we explain how to modify the initial data in order to obtain a limit system that really depends on ν'. We will first present the system, then formally obtain a general limit system that we will validate by choosing unconventional initial data. This limit induces a structure that will enable us to separate the solutions of the initial system into two parts, which we will study separately. The convergence will require new Strichartz estimates.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

May 15th, 2024, 11:30 - 12:15
Seminar room: SR 3.069

Evolving Heterogeneous Elastic Wires

Leonie Langer, University of Ulm

Abstract

The elastic energy of a bending-resistant interface depends both on its geometry and its material composition. We consider such a heterogeneous interface in the plane, modeled by a curve equipped with an additional density function. The resulting energy, which depends on material parameters, captures the complex interplay between curvature and density effects and resembles the Canham–Helfrich functional.

We study a family of planar curves with density evolving in time according to the steepest descent associated to this energy. Describing the curves by their inclination angle, the L^2-gradient flow is a nonlocal coupled parabolic system of second order. We shortly discuss local well-posedness via maximal regularity theory on time-dependent little Hölder spaces. Once global existence is established, convergence of solutions follows with a constrained Łojasiewicz–Simon gradient inequality. We show that the (non)preservation of quantities such as convexity and positivity of the density depends delicately on the choice of material parameters. The same applies for the asymptotic behavior of the system.

This talk is based on joint work with Anna Dall'Acqua (Ulm University), Fabian Rupp (University of Vienna) and Gaspard Jankowiak (University of Graz).

References

[1] A. Dall'Acqua, L. Langer, and F. Rupp. A dynamic approach to heterogeneous elastic wires. *J. Differ. Equ.*, **392** (2024), 1–42.

[2] A. Dall'Acqua, G. Jankowiak, L. Langer, and F. Rupp. Conservation, convergence, and computation for evolving heterogeneous elastic wires. arXiv preprint, accepted for publication in *SIAM J. Math. Anal.*

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

May 15th, 2024, 12:15 - 13:00
Seminar room: SR 3.069

Global Existence and Decay of Small Solutions for Quasi-linear Second-order Uniformly Dissipative Hyperbolic-Hyperbolic Systems

Matthias Sroczinski, University of Konstanz

Abstract

We consider quasilinear systems of partial differential equations consisting of two hyperbolic operators interacting dissipatively. Global-in-time existence and asymptotic stability of strong solutions to the Cauchy problem close to homogeneous reference states are shown in space dimensions larger or equal to 3. The dissipation is characterized by algebraic conditions, previously developed by Freistühler and the speaker, equivalent to the uniform decay of all Fourier modes at the reference state. As a main technical tool para-differential operators are used. The result applies to recent formulations for the relativistic dynamics of viscous, heat-conductive fluids such as notably that of Bemfica, Disconzi and Noronha (2019.).

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May 29th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Energy Barriers for Boundary Nucleation in a Two-well Model

Antonio Tribuzio, Bonn University

Abstract

Shape-memory alloys are specific materials that, e.g. during a cooling process, change their crystalline structure. For this, their internal elastic energy is a multi-well functional acting on the deformation gradient. This transformation is often initiated by the formation of a small inclusion of deformed material (nucleation).

In this talk, we study scaling laws for a double-well singularly-perturbed elastic energy in which the inclusion of deformed material is constraint in the halfspace and has prescribed volume.

This problem is a variant of the isoperimetric problem with an additional (nonlocal and anisotropic) bulk term given by the elastic energy. We will see how the relation between the anisotropy of the material and the constraint affects the scaling.

This is a joint work with Konstantinos Zemas.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

June 5th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Classification of Global Solutions to the Obstacle Problem

Anthony Salib, University of Duisburg-Essen

Abstract

Recently a long standing conjecture on the classification of global solutions to the obstacle problem in dimensions larger than 3 was resolved by Eberle, Figalli and Weiss. Namely, they showed that the zero level set of any global solution to the obstacle problem is an ellipsoid, paraboloid or cylinder with an ellipsoid or paraboloid as base. Although the conjecture was resolved in two dimensions by Sakai in the 80's using complex analysis techniques, his approach leaves closely related problems in two dimensions unresolved. In this talk, I will discuss how the two dimensional result can be obtained without complex analysis as well as some applications of these results.

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June 12th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Convergence of Equilibria of Thin Elastic Plates in a Discrete Model - the von Kármán Case

David Buchberger, University of Augsburg

Abstract

We focus on the mathematical rigorous derivation of von-Karman (vK) plate theory from an atomistic model. We look at the convergence of stationary, possibly non-minimizing points of atomistic energy functionals to the vK-functional. In this sense we are able to extend a recent Gamma-convergence result of Braun and Schmidt under additional assumptions on the derivative of the interaction potential. This talk is based on joint work with Bernd Schmidt.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

June 19th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

On the (In)stability of Several Structures for the Landau-Lifshitz-Gilbert Equation on a Nanowire

Guillaume Ferriere, Inria Lille

Abstract

We are interested in the Landau-Lifshitz-Gilbert equation (LLG) with Dzyaloshinskii-Moriya interaction (DMI) on a nanowire, with or without external magnetic field H_{ext}. This equation features structures connecting $-e_1$ to $+e_1$ (or conversely) called domain walls, which are stationary solutions when $H_{ext} = 0$ and precess explicitly when $H_{ext} = h(t)e_1$. This structure is asymptotically stable, and it is conjectured that generic solutions of LLG decompose into domain walls over time. In this talk, we will focus on two related structures. The first is the 2-domain wall, consisting of two consecutive opposite domain walls. I will show that these structures are also asymptotically stable when initially separated and when H_{ext} drives the domain walls away from each other. This work is in collaboration with Raphaël Côte. The second structure is the stationary solution when $H_{ext} = h_0 e_1$ with constant h_0. I will show that this solution is unstable, but the numerical simulations of the evolution from perturbations of this solution still provide valuable insights regarding the two previous results.

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Nonlinear Partial Differential Equations
SS 24

June 26th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Extensions and Differential Constraints

Franz Gmeineder, University of Konstanz

Abstract

Extension operators are at the core of studying function spaces, allowing us to reduce numerous problems on domains to those on full space. While this theme has witnessed a huge number of contributions over the past century, very little is known on extension operators that preserve certain differential constraints. In this talk, we will give a rather complete picture for divergence-type constraints, where we put a special focus on the borderline case $p = 1$. This is joint work with Stefan Schiffer (MPI MIS Leipzig).

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

July 3rd, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Non-selfadjoint Spectral Problems Related to Self-similar Blowup in Nonlinear Wave Equations

Kaori Nagatou, KIT

Abstract

We consider the wave eqaution with a power nonlinearity

$$u_{tt} - \Delta u = u^p$$

with initial profiles $u(x,0)$ and $u_t(x,0)$, $x \in \mathbb{R}^3$, $t \geq 0$, and $p > 1$ an odd integer. In order to investigate the blowup dynamics we look for radial self-similar blowup solutions of the form

$$u(x,t) = (T-t)^{-\frac{2}{p-1}} U\left(\frac{|x|}{T-t}\right), \qquad T > 0,$$

with a smooth, radial profile U. In particular, we are interested in stability properties of such solutions. This gives rise to analyzing the spectrum of the linearized operator, i.e. to the eigenvalue problem:

$$\mathcal{L}\mathbf{u} = \lambda \mathbf{u},$$

where $D(\mathcal{L}) \subset H^2_{\text{rad}}(B^3) \times H^1_{\text{rad}}(B^3)$, $H^k_{\text{rad}}(B^3) := \{\mathbf{u} \in H^k(B^3) : \mathbf{u} \text{ is radial}\}$,

$$\mathcal{L}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} := \begin{pmatrix} -\rho u_1'(\rho) - \alpha u_1(\rho) + u_2(\rho) \\ u_1''(\rho) + \frac{2}{\rho} u_1'(\rho) - \rho u_2'(\rho) - (\alpha+1) u_2(\rho) + V(\rho) u_1(\rho) \end{pmatrix},$$

$\rho = \frac{|x|}{T-1}$, $V(\rho) = pU(\rho)^{p-1}$ and $\alpha = \frac{2}{p-1}$. We are itnerested in excluding eigenvalues of $\mathcal{L}$ in parts of the right complex half plane, which is ongoing work together with B. Schörkhuber, Y. Watanabe, M. Plum and M.T. Nakao. In this talk, we will show (as a partial result) that all eigenvalues λ in the half-plane $\{\text{Re}(z) > 1 - \alpha\}$ are *real*, which constitutes a substantial advantage for the computation of the desired eigenvalue exclusions. We also provide a global existence region for the complex eigenvalues and upper bounds for the real eigenvalues and show the principle, how we can exclude eigenvalues in such a region.

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July 10th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Two Dimensional Very Weak Solutions to the Monge-Ampère Equation in $C^{1,\frac{1}{3}-}$

Jonas Hirsch, Leipzig University

Abstract

In this informal seminar, I would like to discuss a recent convex integration result on the very weak solutions to the two-dimensional Monge-Ampère equation i.e. we are looking for a function $v\colon \Omega \to \mathbb{R}$, $\Omega \subset \mathbb{R}^2$ be a simply connected domain being a solution to

$$\mathcal{D}etD^2 v = f \text{ in } \Omega \,,$$

where $\mathcal{D}etD^2$ is the distribution, denoted as *very weak Hessian* introduced by Iwaniec, and given by

$$\mathcal{D}etD^2 v = \partial^2_{12}\left(\partial_1 v \partial_2 v\right) - \frac{1}{2}\partial^2_{22}\left((\partial_1 v)^2\right) - \frac{1}{2}\partial^2_{11}\left((\partial_2 v)^2\right) = -\frac{1}{2}\operatorname{curl}\operatorname{curl}(\nabla v \otimes \nabla v)\,.$$

In fact for any $\theta < \frac{1}{3}$ very weak solutions to the two-dimensional Monge-Ampère equation with regularity $C^{1,\theta}$ are dense in the space of continuous functions.

To achieve $\theta < \frac{1}{3}$ a subtle decomposition of the defect at each stage is needed. The decomposition diagonalizes the defect and, in addition, incorporates some of the leading-order error terms of the first perturbation, effectively reducing the required amount of perturbations to one.

I aim to explain why in the setting of the very weak Monge-Ampère equation and of isometric embeddings it is important to reduce the number of spirals/perturbations in each stage as much as possible. If time permits I would like to give an idea of why the above-mentioned diagonalizations allow us to reduce it in the case of Monge-Ampère essentially to one.

This is joint work with Wentao Cao and Dominik Inauen.

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July 17th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Pattern Selection via Marginal Stability of Pushed Fronts in the FitzHugh-Nagumo System

Montie Avery, Boston University

Abstract

Complex coherent structures in physical systems often form after a homogeneous background state becomes unstable. When the transition out of the unstable state is seeded by a spatially localized perturbation, this perturbation grows and forms an invasion front, which propagates into the unstable state and selects a new state in its wake. The marginal stability conjecture asserts that the propagation speed is the unique speed for which the associated invasion front solution is marginally spectrally stable. In many cases, propagation at a fixed speed combines with oscillatory dynamics in either the leading edge or the wake of the invasion process to generate a spatially periodic pattern. Universal wavenumber selection laws predict the wavelength of this pattern through an appropriate combination of the selected speed and the frequency of the temporal oscillations. We explore this phenomenon in the FitzHugh-Nagumo system, a prototypical model for large amplitude pattern formation. In this setting, we give the first rigorous proof of the marginal stability conjecture and associated wavenumber selection laws for any pattern-forming invasion process. This is joint work with Paul Carter and Björn de Rijk.

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Nonlinear Partial Differential Equations
SS 24

July 24th, 2024, 11:30 - 12:30
Seminar room: SR 3.069

Wild Weak Solutions of the 3D Axisymmetric Euler Equations

Patrick Brkic, University of Ulm

Abstract

We consider the Cauchy problem for the 3D incompressible axisymmetric swirl-free Euler equations. The convex integration method developed by De Lellis and Székelyhidi rules out the possibility that the Euler equations admit unique admissible weak solutions. It had remained conceivable, though, that axisymmetry of the solution might serve as a selection criterion. Using a surprising link to the 2D isentropic compressible Euler equations, we will show that this is not the case: There exists initial data for which there are infinetely many admissible swirl-free axisymmetric weak solutions of the 3D incompressible Euler equations. Moreover, we show that there exists an axisymmetric swirl-free initial velocity for which the axisymmetry breaks down instantaneously.