Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 23/24

**October 12th, 2023, 13:00 - 14:00**
**Seminar room: SR 3.061**

# Pushed and Pulled Invasion Fronts in Parabolic PDEs

Matt Holzer, George Mason University

**Abstract**

Invasion fronts refer to fronts that propagate into unstable states. This talk will provide a review of some of the theoretical aspects of invasion fronts and discuss current research efforts. These fronts are often categorized as pushed or pulled and the talk will cover theoretical and numerical aspects of locating these fronts and identifying their speeds as well as their relevance to applications.

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**October 31st, 2023, 11:30 - 13:00**
**Seminar room: SR 3.061**

# On a Compressible Fluid-structure Interaction Problem with Slip Boundary Conditions

Yadong Liu, University of Regensburg

**Abstract**

In this talk, I will present a recent project on a compressible barotropic fluid system interacting with a linear (visco)-elastic solid equation. In particular, the elastic structure formulates the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data. This talk is based on joint work with Sourav Mitra (IIT, Indore) and Šárka Nečasová (IMCAS, Prague).

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**November 7th, 2023, 12:15 - 13:00**
**Seminar room: SR 3.061**

# Stability Threshold of the 2D Couette Flow in a Homogeneous Magnetic Field

Michele Dolce, EPFL

**Abstract**

A planar incompressible and electrically conducting fluid can be described by the 2D Navier-Stokes-MHD system. One simple yet physically relevant laminar state is the Couette flow with a constant homogeneous magnetic field, given by $u_E = (y, 0)$, $B_E = (b, 0)$ in the domain $T \times R$. The goal is to estimate how large can be a perturbation of this state while still resulting in a solution close to the laminar regime, thereby preventing the onset of turbulence. We prove that Sobolev regular initial perturbations of size $O(Re^{-2/3})$, with $Re$ being the Reynolds number, remain close to $u_E$, $B_E$ and exhibit dissipation enhancement. The latter quantifies the convergence towards an x-independent state on a time-scale $O(Re^{-1/3})$, much faster than the standard diffusive one $O(Re^{-1})$.

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**November 7th, 2023, 11:30 - 12:15**
**Seminar room: SR 3.061**

# Shifting Parties in Social Dynamics: a Nonlocal Approach

Olivia Cannon, University of Minnesota

**Abstract**

The bounded confidence model is well-known for its dynamics of party formation within the sphere of social dynamics. We investigate the addition of bias terms, modeling shifts in opinions, and the resulting dynamics including coherent movement of parties. We analyze this movement using a novel, nonlocal approach for the study of the resulting forward-backward delay equations. Different from classical methods, we compute Taylor expansions in function space. This approach leads to an algebraically simple computation of the reduced flow on a center manifold, allowing for proof of coherent small-amplitude movement.

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**November 21st, 2023, 11:30 - 12:00**
**Seminar room: SR 3.061**

# Going Around in Circles: Developing a Functional Analytic Framework for Radial Solutions to PDEs

Dan Hill, University of Saarland

**Abstract**

Understanding spatial localisation in higher dimensions remains an open problem in many contexts. There has been recent progress in studying localised axisymmetric and dihedral patterns by expressing the problem in polar coordinates and utilising theory from radial spatial dynamics. However, such techniques are often insufficient for obtaining rigorous existence results.

In order to further develop analytic tools—such as centre-manifold reductions—for radial PDE systems, we first need to establish the basic theory of radial function spaces. In contrast to general nonautonomous PDEs, radial PDEs possess highly structured nonautonomous terms and explicit smoothness conditions at the origin.

In this talk I will introduce a new framework for radial function spaces, by considering the natural nonautonomous radial differential operators associated with radial PDEs. Following this, I will conclude by briefly discussing future applications of this analytic framework.

This work is in collaboration with Mark Groves (Saarland).

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**November 14th, 2023, 11:30 - 13:00**
**Seminar room: SR 3.061**

# On the Sobolev Stability Threshold for the 2D MHD Equations for vanishing resistivity limit

Niklas Knobel, KIT

**Abstract**

We consider the evolution of the magnetohydrodynamic (MHD) equations in a periodic channel near a combination of Couette flow and constant magnetic field. We study the regime where viscosity is way larger than resistivity. In particular, we show that this regime obtains algebraic growth for vanishing Resistivity. For small data we establish an upper bound on the Sobolev stability threshold.

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**November 21th, 2022, 12:00 - 13:00**
**Seminar room: SR 3.061**

# Breather solutions to quasilinear wave equations

Sebastian Ohrem, KIT

**Abstract**

We consider the quasilinear wave equation

$$V(x)u_{tt} - u_{xx} + \Gamma(x)(u_t^3)_t = 0 \qquad \text{on} \qquad \mathbb{R}\times\mathbb{R}$$

which arises in the study of localized electromagnetic waves modeled by Maxwell equations with Kerr-type optical materials. We are interested in time-periodic, spatially localized solutions, called *breathers*.

For a compactly supported nonlinear potential $\Gamma$ and several choices of the linear potential $V$, we prove existence of breathers using variational methods, and discuss their regularity.

We also consider quasilinear wave equations corresponding to materials of cylindrical geometry or materials with temporally delayed nonlinear response.

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**November 28th, 2023, 11:30 - 13:00**
**Seminar room: SR 3.061**

# Rogue Waves for a Generalized Semilinear Wave Equation

Julia Henninger, KIT

**Abstract**

We study the generalized semilinear wave equation

$$\partial_t^2 u + d(t)M(x, \partial_x)u - |u|^{p-1}u = 0 \quad \text{for } (x,t) \in \mathbb{R}^N \times \mathbb{R}$$

where $M$ is elliptic and $d$ is a positive periodic step potential. Our goal is to construct solutions which are localized in space and time (rogue waves) by means of variational methods. We present our approach with its main difficulties and discuss suitable examples for $M$ and $d$.

This is joint work with Wolfgang Reichel (KIT).

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**December 5th, 2022, 12:30 - 13:00**
**Seminar room: SR 3.061**

# Exponential Decay Estimates of Navier-Stokes System

Shan Wang, UPEC

**Abstract**

We are concerned with the isentropic compressible Navier-Stokes system in the two-dimensional torus, with rough data and vacuum. Arbitrary regions of vacuum are admissible, and no compatibility condition is required. Under these assumptions and for large enough bulk viscosity, global solutions have been constructed by R. Danchin and P.B. Mucha (CPAM, 2023). We establish that these solutions converge exponentially fast to a constant state, and specify the convergence rate in terms of the viscosity coefficients. On the other hand, we prove similar exponential decay results for the solution of the inhomogeneous incompressible Navier-Stokes equations on torus.

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**December 5th, 2022, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Long-wave Approximation of the GP-Hierarchy by the KdV-Hierarchy

Robert Wegner, KIT

**Abstract**

We consider the Gross-Pitaevskii (GP) equation in one dimension, a cubic nonlinear Schrödinger equation with non-zero boundary data $|q| = 1$ at infinity. It is well-known that Schrödinger equations can be transformed into a hydrodynamic form via the so-called Madelung transform. We consider similarly derived variables $(w_+, w_-)$ in which the equation resembles two coupled Korteweg De-Vries (KdV) equations. Under the KdV-scaling

$$W_\pm(T, X) = \frac{1}{\epsilon^2} w_\pm\left(\frac{\sqrt{2}t}{\epsilon^3}, \frac{x}{\sqrt{2}\epsilon}\right) \qquad \epsilon > 0 \text{ small},$$

it has been shown ([Béthuel et al., 2009, 2010]) that $W_+$ and $W_-$ are each approximated by a solution to the KdV-equation in a co-moving frame. We further investigate this connection between (GP) and (KdV). Using the completely integrable structure of both equations, we show that the transmission coefficient, and hence also the higher energies, of (GP), are approximated by those of (KdV). This implies a connection between the respective hierarchies.

## 1 References

[Béthuel et al., 2009] Béthuel, Fabrice and Gravejat, Philippe and Saut, Jean-Claude and Smets, Didier. (2009). On the Korteweg-de Vries long-wave approximation of the Gross-Pitaevskii equation I. *International Mathematics Research Notices*, 2009(14):2700-2748.

[Béthuel et al., 2010] Béthuel, Fabrice and Gravejat, Philippe and Saut, Jean-Claude and Smets, Didier. (2010). On the Korteweg-de Vries long-wave approximation of the Gross-Pitaevskii equation II. *Comm. Partial Differential Equations*, 35(1):113-164.

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**December 19th, 2022, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Optimal Korn-Maxwell-Sobolev Inequalities

Peter Lewintan, KIT

**Abstract**

We present a complete picture of coercive Korn-type inequalities for generalised incompatible fields, and optimally extend and unify several previously known inequalities that are crucial to the existence theory for a variety of models in continuum mechanics.

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**December 12th, 2022, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Computer-assisted Existence Proofs for Nonlinear Boundary Value Problems

Kevin Drescher, KIT

**Abstract**

In this talk we present a computer-assisted method which states existence and local uniqueness of solutions to nonlinear elliptic boundary value problems. Moreover, an enclosure of the solution is given, i.e. we know that there exists a solution in an explicit neighborhood of some approximate solution. The main theorem is based on a fixed-point formulation and in order to satisfy the assumptions of the theorem, some verified numerical computations can be used. This method is illustrated on the basis of a second-order differential equation in divergence form with Dirichlet boundary conditions.

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**January 16th, 2024, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Single- and Multi-Pulses in the Two-Mode Forced Lugiato-Lefever Equation

Lukas Bengel, KIT


**Abstract**

In this talk we consider the two-mode forced Lugiato-Lefever variant

$$iu_t = -du_{xx} + i\omega u_x + (\zeta - i)u - |u|^2 u + i(f_0 + f_1 e^{ikx}) \qquad \text{(LLE)}$$

on the line $\mathbb{R}$. We establish the existence of far-from constant stationary single-pulse solutions, that exhibit small periodic behavior in the asymptotic regions $x \longrightarrow \pm\infty$, and we study their spectral and dynamical stability. In addition, we present a result on the existence of stable multi-pulse solutions of (LLE), which are constructed from superpositions of stable singlepulses. This is joint work with Björn de Rijk.

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**January 23th, 2024, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Asymptotics and Stability of Large $L^\infty$-modulations of Wave Trains in Reaction-diffusion Systems

Joannis Alexopoulos, KIT

**Abstract**

We study reaction-diffusion systems on the extended line admitting a periodic traveling-wave solution $\phi_0$ which is diffusively spectrally stable. Having a solution $u$ and a phase modulation $\gamma$ at hand, we focus on the long-standing question of how the modulated perturbation $u(\cdot,t) - \phi_0(\cdot + \gamma(\cdot,t))$ with respect to suitable norms and initial conditions evolves in time.

I will give an overview of existing answers and present our new result, outlining main aspects and challenges of its proof. The essential extension consists of both lifting any localization requirement on $\partial_x\gamma$ and removing the smallness assumption on $||\gamma(\cdot,t)||_{L^\infty}$. Our method is robust and we expect that it can be applied to other semilinear (even non-parabolic) systems.

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**January 30th, 2024, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Stability of Propagated Fronts in Scalar Balance Laws

Louis Garénaux, KIT

**Abstract**

Scalar balance laws are advection-reaction equations, that appear either in biology or physic when one mesures the variation of a quantity over time. In this presentation, we will focus on special solutions of these equations, namely propagated waves that connect two distinct constant states. A large variety of such waves can be constructed, and we will discuss the stability of some of them. This is joint work with L. M. Rodrigues.

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**February 6th, 2024, 11:30 - 12:30**
**Seminar room: SR 3.061**

# On the Ill-posedness of the Stably Stratified Boussinesq Equations in the Class of Bounded Initial Data

Lars Eric Hientzsch, Bielefeld University

**Abstract**

The Boussinesq equations describe the evolution of a stratified fluid under the influence of gravity. We investigate the system in vorticity form in the setting of a continuous background stratification increasing with depth (spectrally stable density profile). We prove that the system is strongly ill-posed in the class of initial data of bounded vorticity and density gradient. The mechanism that allows us to exhibit the norm-inflation in infinitesimal time is purely nonlinear - in contrast to previous results on mild ill-posedness. Time permitting, applications to the 3D axisymmetric Euler equations with small bounded initial vorticity are discussed. The respective ill-posedness result is based on the norm inflation of the swirl. Based on joint work with R. Bianchini (CNR, IAC Rome) and F. Iandoli (Università della Calabria).

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**February 13th, 2024, 11:30 - 12:30**
**Seminar room: SR 3.061**

# Partially Dissipative Hyperbolic Systems: Hypocoercivity and Hyperbolic Approximations

Timothée Crin-Barat, FAU

**Abstract**

In this talk, we review recent results on so-called partially dissipative hyperbolic systems. Such systems model physical phenomena with degenerate dissipative terms and appear in many applications. For example, in gas dynamics where the mass is conserved during the evolution, but the momentum balance includes a diffusion (viscosity) or a damping (relaxation) term.

First, using tools from the hypocoercivity theory and precise frequency decompositions, we derive sharp stability estimates for linear systems satisfying the Kalman rank condition. This linear analysis allows us to establish new global-in-time existence and large-time behaviour results in a critical regularity framework for nonlinear systems.

Then, we interpret partially dissipative systems as hyperbolic approximations of parabolic systems, in the context of the paradox of infinite speed of propagation. In particular, we focus on a hyperbolic approximation of the multi-dimensional compressible Navier-Stokes-Fourier system and establish its hyperbolic-parabolic strong relaxation limit.