Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

April 18th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

The Validity of the Derivative NLS Approximation for Systems with Quadratic Nonlinearities

Raphael Taraca, Universität Stuttgart

Abstract

The Derivative Nonlinear Schrödinger equation can be derived as an envelope equation via multiple scaling perturbation analysis from dispersive wave systems. It occurs when the cubic coefficient for the associated NLS equation vanishes for the spatial wave number of the underlying slowly modulated wave packet. It is the purpose of this paper to prove that the DNLS equation makes correct predictions about the dynamics of the Klein-Gordon model with a quadratic nonlinearity. The proof is based on energy estimates and normal form transformations. Difficulties occur due to total resonances and second order resonances in the cubic terms and further stable/unstable resonances in the quartic terms. The approximation property holds if the latter resonances are stable. In the unstable case we give a proof of the failure of the DNLS approximation when considering spatially periodic boundary conditions.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

April 25th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Cahn-Hilliard Models with Dynamic Boundary Conditions: Phase Separation Processes and Two-phase Flows

Patrik Knopf, Universität Regensburg

Abstract

The Cahn-Hilliard equation is the most common model to describe phase separation processes in a mixture of two materials. Moreover, it is further used to describe different phenomena where the distribution and/or motion of two (or more) immiscible materials is considered. Standard Cahn–Hilliard models are usually endowed with homogeneous Neumann boundary conditions for both the phase-field variable and the chemical potential. However, these boundary conditions yield certain limitations:

1.) The diffuse interface separating the materials is enforced to intersect the boundary at a perfect angle of ninety degrees, which is unrealistic in many applications.

2.) No transfer of material between bulk and boundary is allowed and thus, absorption process cannot be described. For these reasons dynamic boundary conditions for the Cahn-Hilliard equation have been introduced. We take a closer look at dynamic boundary conditions that also exhibit a Cahn-Hilliard type structure.

To describe the evolution of two-phase flows, Navier-Stokes-Cahn-Hilliard models have become a popular choice. As the standard models are subject to a no-slip boundary condition for the velocity field as well as homogeneous Neumann conditions for the Cahn–Hilliard subsystem, they exhibit the aforementioned limitations and are also not capable of describing moving contact line phenomena. However, these issues can also be overcome by the introduction of suitable dynamic boundary conditions.

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

May 2nd, 2023, 14:00 - 15:30
Zoom Link: https://kit-lecture.zoom.us/j/5732649920
Meeting ID: 573 264 9920
Seminar room: SR 3.068

On Rigorous Derivation of the Hartree Equation from Quantum Many-Body Systems

Ioannis Anapolitanos, KIT

Abstract

One of the reasons that the NLS equation is famous is that it is an approximate model arising from many different physical contexts like magnetohydrodynamics, optics or many-body quantum systems. Often it is assumed that the approximation is good and the properties of NLS itsself are intensely investigated. But how can one prove that the NLS is a good approximation or even to begin with how can one rigorously formulate the question of approximation? In this talk I will explain how to formulate this question for a variant of the NLS equation which is the Hartree equation. I will rigorously formulate the question of approximation in the context of quantum many-body systems. I will then discuss the old results of Pickl and Knowles and Pickl whose arxiv variants can be found in `https://arxiv.org/abs/0907.4464` `https://arxiv.org/abs/0907.4313`. These papers have the simplest existing proof of convergence of many-body Schrödinger dynamics towards the Hartree dynamics. The importance of such a result relies on the fact that the quantum many-body dynamics is, in contrast to the Hartree equation, very far from being numerically computable with todays existing computational power. Purpose of the talk will be an introduction to the subject and is going to partially be based on the attached Bachelor Thesis which presents a simplified version of the above papers.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

May 9th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Free Boundary Problems via Da Prato - Grisvard Theory

Patrick Tolksdorf, KIT

Abstract

A common way to prove global well-posedness of free boundary problems for incompressible viscous fluids is to transform the equations governing the fluid motion to a fixed domain with respect to the time variable. An elegant and physically reasonable way to do this is to introduce Lagrangian coordinates. These coordinates are given by the transformation rule

$$x(t) = \xi + \int_0^t u(\tau, \xi)\, d\tau\,,$$

where $u(\tau, \xi)$ is the velocity vector of the fluid particle at time τ that initially started at position ξ. The variable $x(t)$ is then the so-called Eulerian variable and belongs to the coordinate frame where the domain that is occupied by the fluid moves with time. The variable ξ is the Lagrangian variable that belongs to time fixed variables. In these coordinates the fluid only occupies the domain Ω_0 that is occupied at initial time $t = 0$.

To prove a global existence result for such a problem, it is important to guarantee the invertibility of this coordinate transform globally in time. By virtue of the inverse function theorem, this is the case if

$$\nabla_\xi x(t) = \mathrm{Id} + \int_0^t \nabla_\xi u(\tau, \xi)\, d\tau$$

is invertible. By using a Neumann series argument, this is invertible, if the integral term on the right-hand side is small in $L^\infty(\Omega_0)$. Thus, it is important to have a global control of this L^1-time integral with values in $L^\infty(\Omega_0)$. If the domain is bounded, this can be controlled by decay properties of the corresponding semigroup operators that describe the motion of the linearized fluid equation. On certain unbounded domains, however, these decay properties are not true anymore. While there are technical possibilities to fix these problems if the boundary is compact, these fixes cease to work if the boundary is non-compact.

As a model problem, we consider the case where Ω_0 is the upper half-space. To obtain estimates of the L^1-time integral we establish a homogeneous version of the celebrated theorem of Da Prato and Grisvard (1975) about maximal regularity in real interpolation spaces. In these lectures, we will describe this homogeneous Da Prato-Grisvard theorem in detail and show how it can be applied to solve problems from fluid mechanics involving a free non-compact boundary.

This is a joint work with Raphaël Danchin, Matthias Hieber, and Piotr Mucha.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

May 16th, 2023, 14:45 - 15:25
Seminar room: SR 3.068

Quantitative Quasi-invariance for Gaussian Measures Under the NLS Flow and an Application.

Chenmin Sun, CNRS & University Paris-Est Créteil

Abstract

In the statistical study of Hamiltonian PDEs out of the equilibrium (lack of invariant measures), it is a natural question to understand the transport properties for canonical gaussian measures. In this talk, I will explain the main strategy to prove the quasi-invariance of Gaussian measures at (relatively) high regularities along the 2D NLS flow on the torus, with L^p bounded Radon-Nikodym density (with respect to some weighted Gaussian measure). As an application, we obtain the almost sure multilinear smoothing effect of the cubic NLS on the torus, which cannot be expected for deterministic initial data.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

May 16th, 2023, 14:00 - 14:40
Seminar room: SR 3.068

Vanishing Limit of a Small Solid in a Three-dimensional Incompressible Viscous Fluid.

Jiao He, University Paris-Saclay

Abstract

In this talk, I will present our recent result on the study of the evolution of a small rigid body in an incompressible viscous fluid that filles the whole space $\mathbb{R}^3$. When the small rigid body shrinks to a "massless" point in the sense that its density is constant, we prove that the solution of the fluid-rigid body system converges to a solution of the Navier-Stokes equations in the full space. To achieve this, I will introduce a technique that utilizes L^p-L^q estimates of the fluid-structure semi-group and a fixed-point argument to obtain a uniform estimate of velocity of the rigid body. I then will present the construction of the test function and the process of passing to the limit. This is a joint work with Pei Su (Charles University).

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

June 6th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Features of a Fractional Stokes-Transport system

Dimitri Cobb, Universität Bonn

Abstract

In this talk, we will present two interesting features of a Stokes-transport system with a fractional viscosity law. This system, which is used as a model for understanding the sedimentation of a cloud of particles in a fluid, belongs to a large class of PDEs which are actively studied today: active scalar equations. After presenting the system, we will start by showing how the presence of gravity stratificaion affects the (possible) blow-up of solutions. This will take the form of an anisotropic criterion for detecting blow-up solutions. Secondly, we will see how uniqueness of solutions can be shown in a critical regularity setting. This last part will involve a discussion on the transport equation with non-Lipschitz coefficients.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

June 13th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Anderson localization for random Dirac operators

Sylvain Zalczer, KIT

Abstract

The Dirac operator, originally introduced to describe the motion of a relativistic electron, has been given in the last ten years a lot of attention since it appeared to be a good model for graphene. I will present the results I got with J.-M. Barbaroux (Toulon) and H. D. Cornean (Aalborg), where a random potential is added to the Dirac operator. We prove that, in a simple case, the phenomenon called Anderson localization happens : the disordered material becomes an insulator in a spectral region where it should be conducting. The proof uses a technique called "multiscale analysis", in the version developed by Germinet and Klein.

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

June 20th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Infinitely many Solutions to Generalized Quasilinear Critical Schrödinger Equations in $\mathbb{R}^N$

Laura Baldelli, Polish Academy of Sciences

Abstract

In this talk, we will discuss some recent results concerning quasilinear Schrödinger problems in the entire $\mathbb{R}^N$ of the type

$$-\Delta_p u - \frac{\alpha}{2}\Delta_p(|u|^\alpha)|u|^{\alpha-2}u = \lambda V(x)|u|^{k-2}u + \beta K(x)|u|^{p_\alpha^*-2}u \quad \text{in } \mathbb{R}^N, \tag{1}$$

with $1 < p < N$, $\alpha > 0$, $\max\{1,\alpha\} < k < p_\alpha^* := \max\{p^*, \alpha p^*\}$, V, K nonnegative nontrivial weights and λ, β positive real parameters associated to the subcritical and critical terms, respectively. Quasilinear equations of the type (1) are related to models of several physical phenomena such as plasma physics, high-power ultrashort laser in matter, fluid mechanics, the theory of Heisenberg ferromagnets and magnons.

Our analysis is twofold since the two cases $\alpha > 1$ and $0 < \alpha < 1$ give rise to a different nature of the equation under consideration, indeed, in the first case the term $\Delta_p(|u|^\alpha)|u|^{\alpha-2}u$ is degenerate at $u = 0$, while in the latter case it becomes singular when $u = 0$. For this reason, we will justify the definition of the critical exponent p_α^* in terms of compactness of a suitable embedding and of nonexistence of solutions beyond p_α^*.

Under suitable conditions on the subcritical exponent k, we obtain multiplicity results with negative and positive energy depending on the range of the parameters λ, β. We analyze also the case in which the weights satisfy some symmetry conditions with respect to a certain group $T \subset O(N)$, where $O(N)$ is the group of orthogonal linear transformations in $\mathbb{R}^N$.

Our proofs rely on variational tools, including concentration compactness principles, needed for overcoming the double loss of compactness due both to the critical exponent and to the unboundedness of the domain. In addition, since we cannot deal directly with the energy functional associated with (1) because it might be not well defined, a necessary reformulation of the original problem in a suitable variational setting is performed by using a nice change of variables involving an auxiliary function, delicate to be managed.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 2023

Speaker: Dr. Vincent Duchêne
May 23rd, 2023, 14:00 - 15:30
Seminar room: 3.068

Why and how to rectify a deep water model for surface gravity waves?

CNRS & University Rennes 1

Abstract

We will discuss the so-called "WW2" system which models the propagation of waves at the surface of an infinitely deep layer of fluid, in a small steepness regime. We will first argue that the initial-value problem for this system is most certainly ill-posed in finite-regularity functional spaces, and precisely point out the underlying instability mechanism. Based on this analysis, we will propose a non-invasive regularization procedure which allows to recover all desired properties in terms of modelling and numerical implementation.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

June 27th, 2023, 14:00 - 15:30
Seminar room: SR 3.068
Zoom Link: https://kit-lecture.zoom.us/j/5732649920
Meeting ID: 573 264 9920

A Non-local Free Boundary Problem Arising in a Model of Cell Polarization

Anna Logioti, Universität Stuttgart

Abstract

We consider a model for cell polarization as a response to an external signal which consists of a bulk-surface reaction-diffusion system of equations. We have proved that in a suitable scaling limit the system converges to a non-local free boundary problem. In this talk, I will present several results for this problem, starting with an L1-contraction property and, in the case of time-constant signals, the stability of stationary states. To gain more insight into the evolution of the support of the solution, we further investigate qualitative properties of the free boundary. In particular, we have concluded that there are necessary and sufficient conditions for the initial data that yield continuity of the support at $t = 0$. If one of these assumptions fail, then jumps of the support take place. In addition we have provided a complete characterization of the jumps for a large class of initial data.

This is a joint work with B. Niethammer, M. Röger and J. J. L. Velázquez.

Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

July 4th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Nonlocal-to-local Convergence of Cahn-Hilliard Equations

Lara Trussardi, Universität Konstanz

Abstract

The Cahn-Hilliard equation is widely used in the study of phase field models. A nonlocal version of the equation, proposed by Giacomin and Lebowitz, attracted great interest in recent years. In this talk we show existence and uniqueness of solutions for nonlocal Cahn-Hilliard equations with degenerate potential and present the convergence of a nonlocal version of the Cahn-Hilliard equation to its local counterpart as the nonlocal convolution kernel approximates a Dirac delta in a periodic and Neumann boundary conditions setting. This is based on a series of joint works with E. Davoli, H. Ranetbauer, and L. Scarpa.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

July 11th, 2023, 14:45 - 15:30
Seminar room: SR 3.068

On the continuum limit for discrete Dirac operators on 2D square lattices

Karl Michael Schmidt, Cardiff University

Abstract

The talk discusses the continuum limit of discrete Dirac operators on the two-dimensional square lattice as the mesh size tends to zero. We use the most natural and simplest embedding of the discrete Hilbert space into the continuum Hilbert space, and the question arises naturally when discretising the Dirac operator in two-dimensional Euclidean space, e.g. for numerical analysis. The discrete Dirac operator converges to the continuum Dirac operator in the strong resolvent sense, but not in the norm resolvent sense. The latter result is closely related to the observation that the Liouville theorem does not hold in discrete complex analysis. This is joint work with Tomio Umeda.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

July 11th, 2023, 14:00 - 14:45
Seminar room: SR 3.068

Complete Non-selfadjointness for Schroedinger Operators on the Semi-axis

Ian Wood, University of Kent

Abstract

In this talk we investigate complete non-selfadjointness for all maximally dissipative extensions of a Schrödinger operator on a half-line with dissipative bounded potential. We show that all proper maximally dissipative extensions (that preserve the differential expression) are completely non-selfadjoint. However, it is possible for non-proper maximally dissipative extensions to have a one-dimensional reducing subspace on which the operator is selfadjoint. We give a characterisation of these extensions and the corresponding subspaces and present a specific example. This is joint work with Christoph Fischbacher and Sergey Naboko

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Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 22/23

July 14th, 2022, 15:45 - 16:45
Seminar room: SR 3.068

Formal Modulation Theory for Dynamic Bifurcations in PDEs

Samuel Jelbart, Technical University of Munich

Abstract

This talk focuses on the development of formal modulation theory for dynamic bifurcations in slow-fast PDEs. We show that the formal part of classical modulation theory can be extended to the slow-fast setting using a novel adaption of the so-called geometric blow-up method and the method of multiple scales. We demonstrate the utility and versatility of this approach by using it to derive modulation equations, i.e. simpler closed form equations which govern the dynamics of the formal approximations near the underlying bifurcation point, in the context of model equations with dynamic bifurcations of (i) Turing, (ii) Hopf, (iii) Turing-Hopf, and (iv) stationary long-wave type. The modulation equations have a familiar form: They are of real Ginzburg-Landau (GL), complex GL, coupled complex GL and Cahn-Hilliard type respectively. In contrast to the modulation equations derived in classical modulation theory, however, they have time-dependent coefficients induced by the slow parameter drift, they depend on spatial and temporal scales which scale in a dependent and non-trivial way, and the geometry of the space in which they are posed is non-trivial due to the blow-up transformation. The formal derivation of the modulation equations provides the first steps toward the rigorous treatment of these challenging problems, which remains for future work.

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Nonlinear Partial Differential Equations
SS 23

July 18th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

From Competing Species to Nodal Solutions of the Yamabe Problem

Alberto Saldana, National Autonomous University of Mexico

Abstract

What is the relationship between several populations fighting for resources and the Poincaré conjecture? These two concepts seem to have nothing in common, but in this talk we will establish a close link by relating three mathematical objects: the Yamabe equation, competitive systems (that model the interaction between two species), and optimal partitions of the sphere. This is joint work with Mónica Clapp and Andrzej Szulkin.

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Nonlinear Partial Differential Equations
SS 23

July 20th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Uniqueness results for local and non-local Dirichlet problems

Isabella Ianni, Università di Roma, La Sapienza

Abstract

We present our recent contributions on the problem of the uniqueness for positive solutions of semilinear Dirichlet problems with a power nonlinearity. This question arose from the famous symmetry result by Gidas, Ni, Nirenberg (1979), which implies uniqueness for the Lane-Emden problem when the domain is a ball. A conjecture on the uniqueness in any convex domain was then formulated during the eighties, but only partial answers have been given so far. In this talk we describe recent results for the Lane-Emden problem obtained in dim=2. We also discuss some uniqueness results in the nonlocal case

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SS 23

July 25th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Density Patch Problem for Compressible Fluids

Marcel Zodji, Université Paris Cité

Abstract

The motion of a compressible viscous barotropic fluid is described by the Navier-Stokes system. It is a system of hyperbolic-parabolic mixed-type PDEs. In this talk, we will study the so-called density patch problem: *If we are given a density that is initially discontinuous across a $\mathcal{C}^{1+\alpha}$ curve γ and α-Hölder continuous on the two disjoint components delimited by γ, is this structure preserved in time?*

An important quantity in the mathematical analysis of this system is the so-called effective flux, which was discovered in [Hoff and Smoller, 1985]. More precisely, the mathematical properties of this quantity play a crucial role in the study of the propagation of oscillations in compressible fluids [Serre, 1991], in the construction of weak solutions [Lion, 1996], or the propagation of discontinuity surfaces [Hoff, 2002], to cite just a few examples. In the case of density-dependent viscosities, the behavior of the effective flux degenerates, which renders the analysis more subtle.

1 References

[Hoff, 2002] Hoff, D. (2002). Dynamics of singularity surfaces for compressible, viscous flows in two space dimensions. *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences*, 55(11):1365-1407.

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[Serre, 1991] Serre, D. (1991). Variations de grande amplitude pour la densite d'un fluide visqueux compressible. *Physica D: Nonlinear Phenomena*, 48(1):113-128