Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 25

April 23rd, 2025, 11:30 - 13:00
Seminar room: SR 3.069

On the Well-posedness of the NLS & dNLS Hierarchies

Joseph Adams, Heinrich Heine University Düsseldorf

Abstract

In this talk we will be covering recent local and global well-posedness results related to the higher order nonlinear Schrödinger (NLS) and derivative nonlinear Schrödinger (dNLS) hierarchy equations. As completely integrable systems both the NLS & dNLS equations possess an infinite sequence of conserved quantities, to each of which a generalised Schrödinger equation may be associated.

We begin by explaining the derivation of the general structure of such higher-order equations. For the NLS equation one may then go on to prove multilinearly refined Strichartz estimates with which local well-posedness may derived. Utilising appropriate function spaces for the initial data these well-posedness results can be shown to be optimal up to scaling. Solutions to the equations for initial data in L^2-based spaces can be extended globally using conserved quantities known from the literature.

For the dNLS hierarchy equations more intricate details of the structure of nonlinear terms must be derived. An analogue of the “simplifying” of dNLS using the gauge-transformation has to be found for the higher order equations in order to again derive, up to scaling optimal, local well-posedness results. In some cases solutions may again be extended globally.

We end the talk by giving an overview of related unresolved questions. This talk covers results published in two recent papers of the speaker.

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April 30th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Intrinsic Convexity in the Mullins-Sekerka Evolution

Wenhui Shi, Aachen University

Abstract

The Mullins-Sekerka flow is a curvature driven flow of the interface, where the normal velocity at each point of the interface is a nonlocal quantity of the mean curvature. It appears as the sharp interface limit of the Cahn-Hilliard equation which models the phase transition phenomena, and inherits a gradient flow structure. In this talk, I am going to describe the asymptotic behavior of the Mullins-Sekerka flow around the equilibrium in the full space setting. By exploring the gradient flow structure and the intrinsic convexity properties of the energy, we obtain a refined convergence rate of the energy and energy dissipation. This is based on joint work with Maria Westdickenberg and Michael Westdickenberg.

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May 7th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Global well-posedness of a 2D fluid-structure interaction problem without dissipation

Haocheng Yang, University of Paris-Saclay

Abstract

In this talk, we will analyze the incompressible Euler equation in a time-dependent 2D fluid domain, whose interface evolution is governed by the law of linear elasticity without damping. Our main result asserts that the Cauchy problem is globally well-posed in the energy space for irrotational initial data without any smallness assumption. We also prove the continuity with respect to the initial data and the propagation of regularity. In the absence of parabolic regularization, a key ingredient in our analysis is a novel reduction to a nonlinear Schrödinger-type equation, allowing us to apply dispersive estimates. To carry this out, we develop new estimates for the Dirichlet-to-Neumann operator in low-regularity regimes through tools from classical harmonic analysis and paradifferential calculus. This is a joint work with Thomas Alazard and Chengyang Shao.

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May 14th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Nonliner Fourier Transform Solutions to Nonlinear Long Wave Equations

Patrick Nabelek, Oregon State University

Abstract

When one solves linear wave equations one can use the linear Fourier transform to construct solutions that can be interpreted as linear superpositions of monochromatic waves. With the nonlinear Fourier transform (also called the inverse scattering transform) one constructs solutions to nonlinear completely integrable PDEs as nonlinear superpositions of basic nonlinear waves. In this talk I will discuss the implementation of the nonlinear Fourier transform for various nonlinear long wave equations (Korteweg-de Vries, Kadomtsev-Petviashvili, and Kaup-Broer) using a nonlocal dbar problem (equivalent to a Riemann-Hilbert problem or a function theory problem on a Riemann surface) to take nonlinear superpositions of a kind of nonlinear wave called a soliton. This method is general enough to compute localized solutions, periodic solutions, and undular bores (dispersive shockwaves). I will also discuss the characterization of a wave field via its soliton content.

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May 21th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Steady Bubbles and Drops in Inviscid Fluids

Lukas Niebel, University Münster

Abstract

In this talk we study steady bubbles and drops, which are traveling wave solutions to the axisymmetric two-phase Euler equations with surface tension, whose inner phase is a bounded connected domain. The solutions have a uniform vorticity distribution in this inner phase and they have a vortex sheet on its surface.

Our construction relies on a perturbative approach around an explicit spherical solution, given by Hill's vortex enclosed by a spherical vortex sheet. The construction is sensitive to the Weber numbers describing the flow. At critical Weber numbers, we perform a bifurcation analysis utilizing the Crandall–Rabinowitz theorem in Sobolev spaces on the 2-sphere. Away from these critical numbers, our construction relies on the implicit function theorem. Our results imply that the model containing surface tension is richer than the ordinary one-phase Euler equations, in the sense that for the latter, Hill's spherical vortex is unique (modulo translations) among all axisymmetric simply connected uniform vortices of a given circulation.

This is joint work with David Meyer (ICMAT) and Christian Seis (Münster).

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May 28th, 2025, 11:30 - 12:15
Seminar room: SR 3.069

On the Nonlinear Transverse Asymptotic Stability of Line Solitary Waves for the Three-Dimensional Euler–Poisson System

Changzhen Sun, University of Besançon

Abstract

The ionic Euler–Poisson system serves as a hydrodynamic model for plasma, describing the motion of ions interacting with a background electron density. It is known that this three-dimensional system admits a family of one-dimensional solitary waves that propagate in a single direction with exponential localization.

In this talk, we investigate the stability of these line solitary waves in the three-dimensional Euler–Poisson system. Since perturbations are allowed to depend on the transverse variables, this is referred to as transverse stability. We show that the line solitary waves are nonlinearly transversely stable: solutions starting close to a line solitary wave exist globally in time and converge, in the L^∞_x norm, to a modulated solitary wave as time tends to infinity.

This is joint work with Frédéric Rousset (Orsay, France).

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May 28th, 2025, 12:15 - 13:00
Seminar room: SR 3.069

Global Well-posedness for the Derivative Nonlinear Schrödinger Equation on the Torus

Hajer Bahouri, Sorbonne University

Abstract

Integrable PDEs are regularly used to model a wide range of phenomena observed in nonlinear optics, magnetohydrodynamics and internal waves in fluid mechanics. In this joint work with Galina Perelman, we have been interested by the nonlinear Schrödinger equation with derivative on the Torus. This completely integrable equation which involves in several physical phenomena has been the subject of many works for a long time. The question of global well-posedness in the scale of Sobolev spaces was for a long time restricted to data below the threshold of algebraic solitons. In the early 2000s, this problem was completely solved on the real line. In a recent joint work with Galina Perelman, we have overcome the threshold of algebraic solitons on the Torus. Our proof combines profile decompositions with a refined analysis of the spectral system associated with the equation.

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June 4th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Orbital Stability of a Traveling Wave of the Gross-Pitaevskii Equation

Fédéric Valet, CY Cergy Paris Université

Abstract

The Gross-Pitaevskii (GP) equation in dimension 2 is a model for Bose-Einstein condensates. The solutions have a non-vanishing condition at infinity : the modulus of the solutions goes to 1 at infinity, which makes the analysis different from the usual non-linear Schrödinger equation. Unlike the 1-dimensional case, it is also not known if this non-linear dispersive equation is integrable and the momentum is defined only formally. Due to the competition between the dispersion and the non-linearity, there exist traveling waves moving at a velocity $c \in (0, \sqrt{2})$. In this talk, we investigate a new proof of the orbital stability of traveling waves with small speed using an adequate "quadratic" form. More precisely, we explain how to obtain a suitable definition of the momentum and detail how the non-linearity strongly influences the "quadratic" form at infinity. The talk is based on a collaboration with Philippe Gravejat and Eliot Pacherie (CY Cergy Paris Université).

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June 25th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Schrödinger Equation with Point Interactions: Well-posedness and Standing Waves

Raffaele Scandone, University of Naples

Abstract

We introduce Schrödinger operators with point interactions, a class of L^2-based self-adjoint operators with zero-range potentials. We establish suitable dispersive estimates, and provide an explicit characterization of the adapted Sobolev spaces. These tools enable us to study nonlinear Schrödinger equations (NLS) with delta-like interactions, for which we present results on well-posedness as well as on the existence and qualitative properties of standing waves. Finally, we discuss some connections between point interactions and classical topics in the theory of elliptic partial differential equations.

Based on joint works with F. Boni, G. Dell'Antonio, V. Georgiev, F. Iandoli, A. Michelangeli, D. Noja, A. Olgiati, and K. Yajima.

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July 2nd, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Rigid Lid Limit for Inviscid Shallow Water Equations with a Flat Bathymetry

Benjamin Melinand, Université Paris Dauphine

Abstract

The rigid lid approximation is a common assumption in the oceanographical literature. It can be seen as a low Froude number limit : ocean currents travel slower than water waves. Mathematically speaking it corresponds to a singular limit. We rigorously justify this limit on different shallow water models such as the *abcd* Boussinesq systems or the Green-Naghdi equations with a flat bathymetry. In our setting two small parameters are involved : the shallowness parameter μ and a parameter ϵ that can be interpreted as a Froude number. The rigid lid limit corresponds to the limit $\epsilon \to 0$ and the free surface formally tends to the rest state whereas the velocity of the fluid converges to a solution of the Euler equations. We study this limit for different topologies and provide rates of convergence with respect to ϵ carefully taking care of the dependence with respect to the shallowness parameter.

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July 9th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Deformation of Striped Phases Induced by Localized Impurities

Qiliang Wu, Ohio University

Abstract

We study the effect of algebraically localized impurities on striped phases in spatially extended domains R^n, where $n = 1, 2, 3$. We develop a functional-analytic framework that allows us to cast the perturbation problem as a regular Fredholm problem despite the presence of the essential spectrum, caused by the soft translational mode. For the case $n = 1$, our results establish the selection of jumps in wavenumber and phase, depending on the location of the impurity and the average wavenumber in the system. We also show that, for select locations, the jump in the wavenumber vanishes. Moreover, for the case $n = 2, 3$, we demonstrate that inhomogeneities select a specific translate of the striped patterns and induce algebraically decaying, dipole-type far-field deformations. Phase shifts and leading order terms are determined by effective moments of the spatial inhomogeneity. Farfield decay is proportional to the derivatives of the Green's function of an effective Laplacian. Technically, we use mode filters and conjugacy to an effective Laplacian to establish Fredholm properties of the linearization in Kondratiev spaces. Spatial localization in a contraction argument is gained through the use of an explicit deformation ansatz and a subtle cancellation in Bloch wave space.

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July 16th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Forced Oscillations of a Spring-Mounted Body by a Viscous Liquid

Clara Patriarca, Université Libre de Bruxelles

Abstract

In this talk, we study the periodic motions of the coupled system S, consisting of an incompressible Navier–Stokes fluid interacting with a structure formed by a rigid body subject to undamped elastic restoring forces and torque around its rotation axis. The motion of S is driven by the uniform flow of the liquid, far away from the body, characterized by a time-periodic velocity field, V, of frequency f. We show that the corresponding set of governing equations always possesses a time-periodic weak solution of the same frequency f, whatever $f > 0$, the magnitude of V and the values of physical parameters. Moreover, we show that the amplitude of linear and rotational displacement is always pointwise in time uniformly bounded by one and the same constant depending on the data, regardless of whether f is or is not close to a natural frequency of the structure. Thus, our result rules out the occurrence of resonant phenomena. This is a joint work with D. Bonheure (Université Libre de Bruxelles) and G.P. Galdi (University of Pittsburgh).

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Second Order Gamma-Limit for the Cahn-Hilliard Functional

Pascal Steinke, Bonn University

Abstract

In this talk we are going to introduce the Cahn–Hilliard functional, which is used as a phase-field approximation for the separation of fluids. After a short recap of existing theory for the first-order Gamma-Limit of the functional, we are going to introduce the notion of a second-order Gamma-Limit. The talk will discuss existing results for the second-order Gamma-Limits and present a novel result for the case of a subquadratic potential with Dirichlet data produced in collaboration with F. Colasanto, I. Fonseca and G. Leoni.