Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 22/23

Speaker: Wilhelm Treschow
October 14rd, 2022, 13:30 - 14:00
Seminar room: SR0.019

Embedded eigenvalues for asymptotically periodic ODE systems

Lund University

Abstract

I will discuss persistance of embedded eigenvalues of a certain Schrödinger-type differential operator under perturbations of an asymptotically periodic potential. The studied perturbations are small and belong to a certain Banach space with a specified decay rate, in particular, a weighted space of continuous matrix valued functions. The set of perturbations for which the embedded eigenvalue persists is shown to form a smooth manifold with a specified co-dimension. This is done using tools from Floquet theory, basic Banach space calculus, exponential dichotomies and their roughness properties, and Lyapunov-Schmidt reduction.

In the end, as a way of showing that the investigated setting exists, a concrete example is presented. The example itself relates to a problem from quantum mechanics and represents a system of electrons in an infinite one-dimensional crystal.

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Nonlinear Partial Differential Equations
WS 22/23

October 25th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

An overview of invasion wave-fronts in reaction-diffusion equations

Louis Garénaux, KIT

Abstract

Invasion wave-fronts are a very important structure to model biological phenomenons. Their mathematical study was initiated in 1937 for reaction-diffusion equations, and seems to currently develop along other interesting directions. In this presentation, I will mainly focus on a stability result for the critical FKPP wave-front.

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Nonlinear Partial Differential Equations
WS 22/23

Novermber 8th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

On the Lugiato-Lefever Model for Frequency Combs in a Dual-Pumped Ring Resonator

Elias Gasmi, KIT

Abstract

Kerr frequency combs are described as solutions of the Lugiato-Lefever equation (LLE)

$$\mathrm{i}a_t = -da_{xx} + (\zeta - \mathrm{i})a - |a|^2 a + \mathrm{i}f(x,t), \qquad a \;\; 2\pi\text{-periodic in } x.$$

The LLE is a nonlinear Schrödinger equation with added terms $-\mathrm{i}a$ corresponding to damping and $\mathrm{i}f(x,t)$ corresponding to forcing. From an experimental point of view it is quite attractive to study the generation of Kerr frequency combs by pumping two modes ($k_0 = 0, k_1 \in \mathbb{Z} \setminus \{0\}$) which corresponds to a forcing term $f(x,t) = f_0 + f_1 \mathrm{e}^{\mathrm{i}(k_1 x - \nu_1 t)}$. We will discuss existence results as well as optimality questions with respect to the design of the pump/microresonator.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 22/23

November 15th, 2022, 14:00 - 15:30
Zoom Link: http://kit-lecture.zoom.us/j/5732649920
Meeting Id: 573 264 9920
Seminar room: SR 3.068

Quasilinear Maxwell Equations with Anisotropic Material Laws

Robert Schippa, KIT

Abstract

We consider Maxwell equations with pointwise, fully anisotropic material laws. The characteristic surface is given by the Fresnel surface, which contains conical singularities. We prove Strichartz estimates for Hölder-continuous coefficients, which allows us to solve quasilinear Maxwell equations in the fully anisotropic case for rough initial data.

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November 22nd, 2022, 14:00 - 15:30
Seminar room: SR 3.068

The Madelung Equations: Interpretations and Well-posedness

Robert Wegner, KIT

Abstract

Under the Madelung transform $q \mapsto (|q|^2, Im(\frac{\partial_x q}{q}))$ the Schrödinger equation transforms into a system of "quantum" Euler Equations, often called the "Madelung equations". This hydrodynamic formulation has historically been an inspiration to physicists searching for a more classical approach to quantum mechanics. We sketch some of these ideas and investigate how a well-posedness result for NLS can be transferred to the Madelung equations.

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Speaker: Rebekka Zimmermann
November 29th, 2022, 14:00 - 15:30
Seminar room: 3.068

Strichartz estimates for Schrödinger equations with critical singularities

KIT

Abstract

We consider Schrödinger equations with real-valued potentials exhibiting strong singularities at the origin. Under suitable assumptions on the potential one can prove uniform weighted resolvent estimates for the Schrödinger operator and we will sketch the main steps of how to use these to prove global-in-time Strichartz estimates. The results are taken from the article "Uniform resolvent and Strichartz estimates for Schrödinger equations with critical singularities" by J. M. Bouclet and H. Mizutani.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 22/23

Speaker: Dr. Ángel Castro
November 29th, 2022, 14:45 - 15:30
Seminar room: 3.068

Global structures for 2D Euler and SQG

Instituto de Ciencias Matemáticas (CSIC)
Madrid

Abstract

In this talk we present some results about the existence of global structures for 2D Euler and SQG. In particular, we discuss the existence of traveling waves close to shear flows and review the existence of rotating solutions for 2D Euler and SQG. Some aspects about the stability of these solutions will also be presented.

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Nonlinear Partial Differential Equations
WS 22/23

December 6th, 2022, 14:00 - 15:30
Seminar room: 3.068

On Echo Chains in Magnetohydrodynamics

Niklas Knobel
niklas.knobel@kit.edu
Karlsruhe Institut of Technology

We consider the evolution of the magnetohydrodynamic (MHD) equations with magnetic dissipation in a periodic channel near an affine flow and a constant magnetic field. Here we, in particular, aim to capture resonances between high and low frequency perturbations, which are known as echoes. More precisely, we construct explicit low frequency waves and study high frequency (chains of) resonances and resulting norm inflation in the linearized problem around these waves. The magnetic field here is shown to have a large effect on the behavior of resonances as compared to the Euler setting.

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December 13th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

Large Sets Without Fourier Restriction Theorems

Constantin Bilz, KIT

Abstract

Fourier restriction inequalities associated to submanifolds form an important area of research in harmonic analysis and are connected to various other problems in mathematics. About 25 years ago, Mockenhaupt and Mitsis showed that the restriction of the Fourier transform to a fractal (instead of a submanifold) can also behave interestingly. But the class of fractals is extensive and necessary conditions for Fourier restriction inequalities in this setting are not well-understood. In this direction, we will show that all Fourier restriction sets avoid a universal set of full Hausdorff dimension.

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Nonlinear Partial Differential Equations
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December 20th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

Computer-assisted Proofs and Eigenvalue Bounds

Kevin Drescher, KIT

Abstract

In this talk a computer-assisted method for nonlinear elliptic boundary value problems, which gives not only the existence and local uniqueness of solutions, but also an enclosure of these, is presented. We consider a weak formulation of second-order differential equations in divergence form with Dirichlet boundary conditions. To apply this method, bounds to the essential spectrum and to isolated eigenvalues of some selfadjoint operator have to be computed. In the lecture we focus on the computation of such eigenvalue bounds, i.e. upper and lower bounds forming sharp enclosures for the eigenvalues. For this purpose, theorems for eigenvalue problems with bilinear forms are presented. Some simple examples illustrate the results.

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January 10th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

Spectral stability of shock profiles for hyperbolically regularized systems of conservation laws

Johannes Bärlin[1], Universität Konstanz

Abstract

This talk reports a proof that under natural assumptions shock profiles viewed as heteroclinic travelling wave solutions to a hyperbolically regularized system of conservation laws of the form

$$g(v)_t + f(v)_x = B\Box v := B(v_{xx} - v_{tt}) \quad (v \in \mathbb{R}^n)$$

are spectrally stable if the shock amplitude is sufficiently small. This means that an associated Evans function $\mathcal{E} : \Lambda \to \mathbb{C}$ with $\Lambda \subset \mathbb{C}$ an open superset of the closed right half plane $\mathbb{H}^+ \equiv \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda \geq 0\}$, has only one zero, namely a simple zero at 0. The result is analogous to the one obtained in [1] and [2] for parabolically regularized systems of conservation laws, and also distinctly extends findings on hyperbolic relaxation systems in [2], [3], [4].

References

[1] H. Freistühler and P. Szmolyan. Spectral stability of small shock waves. *Arch. Ration. Mech. Ana.*, 164: 287–309, 2002.

[2] R. Plaza and K. Zumbrun. An Evans function approach to spectral stability of small-amplitude shock profiles. *Discrete Contin. Dyn. Syst.*, 10(4): 885–924, 2004.

[3] C. Mascia and K. Zumbrun. Spectral stability of weak relaxation shock profiles. *Comm. Partial Differential Equations*, 34(1-3):119–136, 2009.

[4] Y. Ueda. Stability of travelling wave solutions to a semilinear hyperbolic system with relaxation. *Math. Methods Appl. Sci.*, 32(4):419–434, 2009.

[1]This research has been supported by Deutsche Forschungsgemeinschaft under Grant No. FR822/10-1.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 22/23

January 17th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

Existence and Regularity of Breathers for a Quasilinear Wave Equation

Sebastian Ohrem, KIT

Abstract

In this talk we consider a quasilinear wave equation of the form

$$V(x)u_{tt} + u_{xx} + \Gamma(x)(u_t^3)_t = 0, \qquad x \in \mathbb{R}, t \in \mathbb{T}$$

with $\Gamma = \chi_{[-L,L]}$ and $V \geq 0$ on $[-L, L]$, $V < 0$ on $\mathbb{R} \setminus [-L, L]$.

First, for specific choices of V we construct breather solutions using variational methods, which turn out to be distributional solutions.

Second, we want to show that these solutions are weak solutions. We present an approach which yields improved regularity, but not the desired result as of now. This is ongoing research.

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Nonlinear Partial Differential Equations
WS 22/23

January 24th, 2023, 14:00-15:30
Seminar room: SR 3.068

Existence and stability for an extended Lugiato-Lefever equation

Lukas Bengel, KIT

Abstract

In applications, locking of the repetition rate of a Kerr soliton comb inside a microresonator by pumping two modes is of particular interest. Mathematically this can be described by a new variation of the Lugiato-Lefever equation (LLE) given by

$$\mathrm{i}u_t = -du_{xx} + \mathrm{i}V(x)u_x + (\zeta - \mathrm{i})u - |u|^2u + \mathrm{i}f,$$

which is a damped and driven nonlinear Schrödinger equation with an additional potential $V(x)$. In the first part we discuss the existence of nontrivial stationary 2π-periodic solutions of the LLE using bifurcation theory and show that localized solitons can be found if the potential $V(x)$ has a sign change. In the second part we discuss stability properties of these solutions and show numerical simulations with `pde2path` that complement our analytical findings.

The talk is based on ongoing work with Dmitry Pelinovsky, and Wolfgang Reichel.

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Nonlinear Partial Differential Equations
WS 22/23

January 31st, 2022, 14:00 - 15:30
Seminar room: SR 3.068

Variational Methods for a Generalized Semilinear Wave Equation

Julia Henninger, KIT

Abstract

We study the equation

$$\left(\partial_t^2 + d(t)(-\partial_x^2 + \alpha x^2)^2\right) u = d(t)|u|^{p-1}u \quad \text{on } \mathbb{R}^2, \tag{1}$$

where $p \in (1, \infty)$, $\alpha > 0$ and d is a positive periodic step potential. By using variational methods we look for non-trivial solutions of (1). The main difficulty is to verify a spectral condition of the linear part of (1) to obtain a suitable functional analytic setting for our variational method. Then one can find solutions of (1) as a critical point of an appropriate functional. This is ongoing research.

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WS 22/23

February 07th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

(Modulated) Stability of Wave Trains Against Non-localized Perturbations

Joannis Alexopoulos, KIT

Abstract

Our general research goal is to show stability of semilinear differential equations around wave trains. By translational invariance of the linearization, and since we are considering perturbations which need not admit integrability or periodicity conditions, different aspects arise and complicate the conclusion of stability.

My talk will cover an illustration of some of these issues with the help of elementary examples. Furthermore, I will present how we aim to resolve them within our research framework, where we are currently interested in the FitzHugh-Nagumo system.

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Seminar of the Work Group
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WS 22/23

February 14th, 2022, 14:00 - 15:30
Seminar room: SR 3.068

Validity of the Whitham Approximation for a Complex Cubic Klein-Gordon Equation

Sarah Hofbauer, University of Stuttgart

Abstract

The complex cubic Klein-Gordon (ccKG) equation possesses a family of periodic traveling wave solutions. Whitham's modulation equations (WME) can be derived by a multiple scaling perturbation analysis in order to describe slow modulations in time and space of these traveling wave solutions. We prove estimates between true solutions of the ccKG equation and their associated WME approximation. The bounds are obtained in Gevrey spaces and hold independently of the spectral stability of the underlying traveling wave solutions. The proof is based on a suitable choice of variables, Cauchy-Kovalevskaya theory, infinitely many near identity changes of variables, and energy estimates in Gevrey spaces. The analysis for the ccKG equation is more complicated than the analysis for the nonlinear Schrödinger (NLS) equation which has been handled in the existing literature, due to additional curves of eigenvalues leading to an additional oscillatory behavior. This talk is based on a joint work with Xian Liao and Guido Schneider.

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Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 22/23

February 16th, 2022, 14:00 - 15:30
Seminar room: SR 1.067

Normalized Ground States of Nonlinear Schrödinger Equations

Jaroslaw Mederski, Institute of Mathematics, Polish Academy of Sciences

Abstract

We present a simple minimization method to show the existence of normalized ground state solutions to the nonlinear Schrödinger equation

$$\begin{cases} -\Delta u + \lambda u = g(u) & \text{in } \mathbb{R}^N,\ N \geq 3 \\ u \in H^1(\mathbb{R}^N)\,, \\ \int_{\mathbb{R}^N} |u|^2\,dx = \rho > 0\,, \end{cases}$$

where ρ is the prescribed mass. Our approach is based on the direct minimization of the energy functional on a suitable constraint. A crucial step is the application of the profile decomposition theorem involving a general Sobolev-subcritical nonlinearity.

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Seminar of the Work Group
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February 17th, 2022, 14:00 - 15:30
Zoom Link: https://kit-lecture.zoom.us/j/5732649920
Meeting ID: 573 264 9920
Seminar room: SR 2.066

First Steps in Twisted Rabinowitz-Floer Homology

Yannis Bähni, Universität Augsburg

Abstract

Rabinowitz-Floer homology is the Morse-Bott homology in the sense of Floer associated with the Rabinowitz action functional introduced by Kai Cieliebak and Urs Frauenfelder in 2009. In our work, we consider a generalisation of this theory to a Rabinowitz-Floer homology of a Liouville automorphism. As an application, we show the existence of noncontractible periodic Reeb orbits on quotients of symmetric star-shaped hypersurfaces. In particular, our theory applies to lens spaces.