Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 25/26

November 5th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

On the Asymptotic Stability Problem for Soliton Solutions to the Boussinesq-type Models

Christopher Maulén, Bielefeld University

Abstract

In this talk, we introduce the Boussinesq family of equations, revisit some known results, and present new findings on the asymptotic stability of solitary wave solutions in the energy space. In particular, we consider the one-dimensional Kaup–Broer–Kuperschmidt (KBK) model with initial data in the energy space $H^1 \times L^2$. This model belongs to the broader family of `abcd` Boussinesq models introduced by Bona, Chen, and Saut, to describe shallow water waves under the influence of dispersion and large amplitudes. The KBK model admits solitary waves with speeds $c \in (-1, 1)$. Angulo established their orbital stability in $L^2 \times H^1$, assuming local well-posedness in $H^1 \times H^2$. Building on this, we prove that KBK solitary waves are asymptotically stable for initial data in the energy space and for a range of speeds, relying on a new set of virial estimates specifically adapted to the KBK system in a moving frame. This talk is based on a series of joint works with Claudio Muñoz.

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November 12th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Dissipative Euler Flows Originating from Circular Vortex Filaments

Antonio Hidalgo, MPI Leipzig

Abstract

In this talk, we will review some results and heuristics of vortex filaments in the 3D Euler and Navier-Stokes equations. Then, we will show the existence of Euler solutions with a strict vortex filament as initial data. More specifically, we will use convex integration to prove the existence of infinitely many weak solutions to the 3D Euler equations with initial vorticity supported on a circumference. We are able to impose some nice properties on our solutions such as having compactly supported vorticity and decaying energy. This is a joint work with Francisco Gancedo and Francisco Mengual.

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Institute for Analysis
WS 25/26

Speaker: Louis Garénaux
November 19th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Orbital Stability for Monostable Fronts in Reaction-diffusion Equations

Louis Garénaux

Abstract

In this presentation, I will discuss the long-time asymptotic stability of a particular class of traveling waves. Monostable fronts are heteroclinic connections between linearly stable and linearly unstable constant equilibria. Until now, all known stability result assume additional localization of the initial perturbation. Following an idea developed in [Garenaux, Rodrigues: 2025] we will see how to remove this condition to show orbital stability with respect to phase shift. This is work in progress.

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December 3rd, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Orbital Stability of Cnoidal Waves against Localized Perturbations

Emile Bukieda, KIT

Abstract

We investigate the stability and long-term behavior of so-called *cnoidal* waves in the Korteweg de Vries (KdV) and Nonlinear Schrödinger equation (NLS) against localized perturbations. These periodic solutions correspond to shallow water waves and periodic light pulses in nonlinear optical fibers, respectively. To date, nonlinear stability results for periodic waves in Hamiltonian systems have primarily addressed co-periodic or subharmonic perturbations. Their stability with respect to localized perturbations - a natural setting in many physical applications - remains a longstanding open problem, as such perturbations render the wave neither localized nor periodic, placing its stability analysis outside the scope of the classical orbital stability framework for Hamiltonian systems developed by Grillakis, Shatah, and Strauss. We present an alternative approach that combines variational techniques, leveraging conserved quantities tailored to the perturbation equation, with Duhamel-based methods and a modulational ansatz - tools known from the stability analysis of periodic waves in dissipative systems and reaction diffusion models. Using this approach, we establish orbital stability of cnoidal waves with respect to L^2-localized perturbations.

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December 10th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Validity of the NLS Approximation for the Peregrine Soliton Family

Nils Thorin, Stuttgart University

Abstract

In this talk I focus on extending the validity of the Nonlinear Schrödinger (NLS) approximation into various functional spaces that naturally occur when studying solutions of the NLS. We consider a one parameter family of solutions that include the Peregrine soliton(s), Kuznetsov-Ma solitons and Akhmediev breathers, none of which are covered by current approximation results. These have been regarded as possible explanations for phenomena such as rouge waves in water wave systems, and high energy optical pulses in optical systems. The main mathematical difficulty lies in choosing the right functional spaces to do the approximations in. Showing the validity allows us to find this behaviour in more complex systems, and starkly reduce the dimension of numerical computations.

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December 17th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Vanishing Viscosity Non-unique Solutions to the Forced 2D Euler Equations

Giulia Mescolini, EPFL

Abstract

In the last decades, different techniques were developed to prove results around the topic of (non-)uniqueness of fluid dynamical PDEs. It is then an important question to understand if there is a selection principle for these equations, namely if such non-unique solutions can also be obtained in the limit of regularised problems (in which, for instance, a dissipative term is introduced: the vanishing viscosity limit). Remarkably, in the context of conservation laws, selection happens.

In a joint work with Dallas Albritton and Maria Colombo, we address this issue in the context of the forced 2D Euler equations, for which Vishik (2018) recently constructed non-unique solutions. We prove that the unique Leray solutions to the 2D Navier-Stokes system, forced with Vishik's force and starting from a perturbation of Vishik's initial datum, converge to a one-parameter family of solutions when we take the double limit of vanishing viscosity and perturbation size.

For the latter, we discover a uniqueness threshold below which the vanishing viscosity solution is unique and radial, and at which the viscous solutions converge to non-unique, non-radial solutions.

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November 26th, 2025, 11:30 - 13:00
Seminar room: SR 3.069

Rogue Waves for a Semilinear Wave Equation

Julia Henninger, KIT

Abstract

We consider the semilinear wave equation

$$V(x)\partial_t^2 u + d(t)M(x, \partial_x)u = V(x)\tilde{d}(t)|u|^{p-1}u, \quad (x,t) \in \mathbb{R}^N \times \mathbb{R}, \tag{$\star$}$$

where M is in general an elliptic operator and $p \in (1, p^*)$. Under suitable conditions on M and on the coefficients V, d and $\tilde{d}$, we show the existence of solutions which are localized in space and in time, so-called rogue waves, by means of variational methods. We discuss our approach in detail and provide examples of $M, V, d, \tilde{d}$ which satisfy our assumptions.

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January 7th, 2026, 11:30 - 13:00
Seminar room: SR 3.069

Stability of Soliton for Nonlinear Non-integrable Schrödinger Equations with Non-trivial Far Field

Jordan Berthoumieu, Strasbourg University

Abstract

Many innovative technologies in areas such as optical fibers, superfluids and Bose-Einstein condensates are based on quantum physical models described by nonlinear Schrödinger equations. Our main focus is put on the nonlinear Schrödinger equation with non-zero limit at infinity that is a generalization of the well-known Gross-Pitaevskii equation (GP) with a non-vanishing condition at infinity. The presentation will relate to defocusing nonlinearities for which we are specially interested and on the properties of travelling waves. Many different behaviors for these travelling waves have been highlighted, according to the shape of the non-linearity. Nevertheless, we have been able to prove the existence of travelling waves with small momentum. Moreover, we shall dwell on the existence and uniqueness travelling waves with speed close to the speed of sound, the orbital stability of a well-prepared chain of such travelling waves, as well as the asymptotic stability of these special solutions.

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January 14th, 2026, 11:30 - 13:00
Seminar room: SR 3.069

Bright and Dark Pulses in the Lugiato-Lefever Equation with Periodic Forcing

Lukas Bengel, KIT

Abstract

We consider a damped and periodically driven nonlinear Schrödinger (NLS) equation,

$$iu_t = -du_{xx} + icu_x + (\zeta - i)u - |u|^2 u + if(x)$$

which is a variant of the Lugiato-Lefever equation. This model arises in the description of microcombs in nonlinear optics. In the focusing case $d > 0$, we show that periodic forcing leads to stable, localized multi-pulse solutions with oscillatory tails that bifurcate from the bright NLS soliton. For the defocusing case $d < 0$, we prove existence results of dark pulse solutions, which we obtain by pasting together a front and a back solution. Numerical simulations with pde2path corroborate our analytical findings. This is based on a joint project with Björn de Rijk.

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January 21st, 2026, 11:30 - 13:00
Seminar room: SR 3.069

Spectral Asymptotics of Asymptotically Periodic Sturm-Liouville Operators

Sebastian Ohrem, KIT

Abstract

We study the Sturm-Liouville eigenvalue problem

$$-u'' = \lambda V(x) u \qquad \text{for} \quad x \in \mathbb{R}$$

where the potential V is uniformly positive, bounded, and piecewise $W^{1,1}$.

First, if V is additionally periodic, we calculate the asymptotic shape of the spectrum as $\lambda \to \infty$ using modified Prüfer angles. We focus on the case where V has exactly one discontinuity per periodicity cell.

Second, if V is only periodic outside some compact interval, we show that the number of eigenvalues in each gap of the essential spectrum is uniformly bounded.

This allows us to extend the class of examples in the paper [J.Henninger, O., W.Reichel; arXiv:2505.13336] where we constructed time-periodic, space-localized solutions to the semi-linear wave equation

$$-u_{xx} + V(x) u_{tt} = \Gamma(x) |u|^{p-1} u = 0 \qquad \text{for} \quad x, t \in \mathbb{R}$$

with suitable Γ.

This is based on ongoing work together with Michael Plum.

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January 28th, 2026, 11:30 - 13:00
Seminar room: SR 3.069

Computer-assisted Analysis for PDEs on $\mathbb{R}^n$

Matthieu Cadiot, École Polytechnique Paris

Abstract

In this talk I will present a computer-assisted approach for constructively proving the existence of localized solutions in PDEs on $\mathbb{R}^m$. The proof is based on the construction of an approximate solution u_0 with support on a large hypercube using Fourier series, combined with a Newton-Kantorovich approach yielding the existence of a true solution in a vicinity of u_0. Tight Fourier analysis estimates are derived to close the fixed point argument.

Furthermore, we are able to control the spectrum of the linearization around the proven solution. This allows to conclude about spectral (and sometimes nonlinear) (in)stability. Applications to the 2D Swift-Hohenberg PDE, to the 2D Gray Scott model and to the Whitham equation will be presented.

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February 4th, 2026, 11:30 - 13:00
Seminar room: SR 3.069

On the NLS Hierarchy with Nonzero Boundary Condition

Robert Wegner, KIT

Abstract

The NLS and KdV equations are the first infinite dimensional Hamiltonian systems that were proven to be completely integrable. A consequence is that they both have an associated infinite hierarchies of PDEs which generate higher symmetries – the NLS and KdV hierarchies.

We review the literature on well-posedness results for these hierarchies, and present a new result for the NLS hierarchy in the setting of the nonzero boundary condition $\lim_{x\to\pm\infty} q(x) = q_\pm$, where $|q_\pm| = 1$.

Subsequently, we consider the behavior of long-wave solutions to this hierarchy from the perspective of the hydrodynamic variables $w_\pm = \frac{1}{2}\Im[q^{-1}q_x] \pm (|q| - 1)$, and discover that the effective dynamics are given by the KdV hierarchy.

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February 11th, 2026, 11:30 - 13:00
Seminar room: SR 3.069

Discontinuous Solutions of the 2D Compressible Navier-Stokes Equations with Variable Viscosity

Rebekka Zimmermann, KIT

Abstract

In this talk, we consider the two-dimensional compressible Navier–Stokes equations, which model the evolution of a compressible fluid over time. The initial density is assumed to be regular on both sides of a regular curve, with possible discontinuities across this curve. This setting describes, for example, a mixture of two immiscible fluids. The goal is to prove the propagation of the piecewise regularity of the density function, as well as the regularity of the curve of discontinuities. In the first part of the talk, we discuss the general proof strategy found in the literature for the constant-viscosity case. In the second part, we present our result on regularity propagation in the variable-viscosity case and explain some of the underlying proof ideas.

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February 18th, 2026, 11:30 - 13:00
Seminar room: SR 3.069

Stability of Discontinuous Shock Profiles in General Relaxation Systems

Joannis Alexopoulos, KIT

Abstract

The overall goal of the project I report about is to advance the nonlinear stability theory on discontinuous shock profiles for relaxation systems with scalar equilibrium equations due to Yang and Zumbrun. Their theory paradigmatically applies for the inviscid St. Venant equations. Our main extension allows for nonlinear stability whenever the equilibrium description is **not necessarily scalar** and may therefore be relevant for further important physical considerations. In my talk, I introduce and discuss central concepts, such as the "Chapman-Enskog view" or the Evans-Lopatinsky condition, which are the building blocks of our extended stability theory.

This is joint work in progress with Johannes Bärlin, Zhao Yang and Kevin Zumbrun.