Karlsruhe Institute of Technology

Ground states of a nonlinear curl-curl problem

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Institute for Analysis

The problem

$$(*) \qquad \nabla \times \nabla \times U + V(x)U = \Gamma(x)|U|^{p-1}U \quad \text{in} \quad \mathbb{R}^3$$

with $p > 1$, $V \in L^\infty(\mathbb{R}^3)$, $U \in H^1(\mathbb{R}^3; \mathbb{R}^3)$?

Question: does $(*)$ have a ground state?

Outline:

1. Physical background
2. Previous results/our results
3. Some details

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$$\nabla \times E + \partial_t B = 0, \qquad \nabla \cdot D = 0,$$
$$\nabla \times H - \partial_t D = 0, \qquad \nabla \cdot B = 0.$$

Material laws:

$$B = \mu_0 H, \quad D = \epsilon_0 E + P(x, E) = \epsilon_0 (1 + \chi_1(x) + \chi_3(x)|E|^2) E$$

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Quasilinear wave-equation for E:

$$\nabla \times \nabla \times E + \mu_0 \partial_t^2 \Big(\epsilon_0 (1 + \chi_1(x)) E + \epsilon_0 \chi_3(x) |E|^2 E \Big) = 0$$

Goal: time periodic, real-valued, spatially localized solutions

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Goal: time periodic, ~~real-valued~~, spatially localized solutions
Ansatz: $E(x,t) = U(x) e^{i\omega t}$ leads to

$$\nabla \times \nabla \times U \underbrace{- \frac{\omega^2}{c^2} \Big(\underbrace{1 + \chi_1(x)}_{= n^2(x)} \Big)}_{V(x) \le 0} U = \underbrace{\frac{\omega^2}{c^2} \chi_3(x)}_{=: \Gamma(x)} |U|^2 U \text{ in } \mathbb{R}^3$$

Results - Part I

0) $U(x_1, x_2, x_3) = \begin{pmatrix} 0 \\ 0 \\ u(x_1, x_2) \end{pmatrix}$ leads to NLS-problem (many results!)

$$-\Delta u + V(x)u = \Gamma(x)|u|^{p-1} \text{ in } \mathbb{R}^2$$

1) Benci-Fortunato (2004) & Azzollini-Benci-d'Aprile-Fortunato (2006) & d'Aprile-Siciliano (2011):

$$\nabla \times \nabla \times U = W'(|U|^2)U \text{ in } \mathbb{R}^3$$

Existence of ground-states in subspaces of cylindrical symmetry

2) Bartsch-Mederski (2014):

$$\nabla \times \nabla \times U + \lambda U = \partial_U F(x, U) \text{ in } \Omega, \quad \nu \times U = 0 \text{ on } \partial\Omega.$$

3) Results by Bartsch-Dohnal-Plum-R. (2014) for

$$\nabla \times \nabla \times U + V(x)U = \Gamma(x)|U|^{p-1}U \quad \text{in} \quad \mathbb{R}^3$$

... next

Results - Part II

$$(*) \qquad \nabla \times \nabla \times U + V(x)U = \Gamma(x)|U|^{p-1}U \quad \text{in} \quad \mathbb{R}^3$$

Generaly assumption: $V = V(r, x_3), \Gamma = \Gamma(r, x_3), r = \sqrt{x_1^2 + x_2^2}$

Theorem (Defocusing case)

- $\Gamma(x) \leq -C(1 + |x|^\alpha), \alpha > \frac{3}{2}(p-1), p > 1,$
- $V \in L^\infty(\mathbb{R}^3)$, $\sup V < 0$.

Then $(*)$ has a (restricted) ground-state.

Theorem (Focusing case)

- $\inf \Gamma > 0$, $V, \Gamma \in L^\infty(\mathbb{R}^3)$ *are ω-periodic in x_3,*
- $1 < p < 5$
- $0 \notin \sigma(L)$ *with* $L = \nabla \times \nabla \times + V(x)$.

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Variational set-up and symmetries

$$J[U] = \int_{\mathbb{R}^3} \frac{1}{2}|\nabla \times U|^2 + \frac{V(x)}{2}|U|^2 - \frac{\Gamma(x)}{p+1}|U|^{p+1}\,dx, \quad U \in H^1(\mathbb{R}^3;\mathbb{R}^3)$$

Here is the problem: $\|\nabla U\|_{L^2}^2 = \|\nabla \times U\|_{L^2}^2 + \|\nabla \cdot U\|_{L^2}^2$.

Constraint $\{U : \operatorname{div} U = 0\}$ does not solve it $\Rightarrow$ Lagrange-multiplier!

Symmetries will help:

Suppose $\Gamma(x) = \Gamma(Mx)$, $V(x) = V(Mx)$ for $M \in O(3)$.

If $V(x) := M^T U(Mx)$ then $J[V] = J[U]$

Variational symmetry $\Rightarrow$ symmetry of PDE

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Radial symmetry

Functions "depending only" on $r = |x|$
(compare with NLS: $-\Delta u + V(x)u = \Gamma(x)|u|^{p-1}u$)

NLS case:

$$\begin{aligned} u(x) =& u(Mx) \\ & \forall x \in \mathbb{R}^n, \forall M \in O(n) \\ \Rightarrow u(x) =& w(|x|) \end{aligned}$$

Differential equation:

$$-(w'' + \frac{n-1}{r} w') + V(r)w = \Gamma(r)|w|^{p-1}w$$

$$H^1_{O(n)} \hookrightarrow L^q(\mathbb{R}^n), 2 \le q < \tfrac{2n}{n-2}$$

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Curl-Curl case:

$$\begin{aligned} U(x) =& M^T U(Mx) \\ & \forall x \in \mathbb{R}^3, \forall M \in O(3) \\ \Rightarrow U(x) =& f(|x|)\frac{x}{|x|} \end{aligned}$$

"Differential" equation:

$$V(r)f(r) = \Gamma(r)|f(r)|^{p-1}f(r)$$

no interesting solutions

Cylindrical symmetry

Functions "depending only" on (r, x_3) with $r^2 = x_1^2 + x_2^2$

Curl-Curl case: $U(x) = M^T U(Mx) \quad \forall x \in \mathbb{R}^3, \forall M \in G_0$ where

$$G_0 = \left\{ \begin{pmatrix} cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} : \alpha \in [0, 2\pi) \right\}$$

Azzollini-Benci-d'Aprile-Fortunato: $U = Q + S + T$ with

$$Q = \frac{q(r, x_3)}{r} \begin{pmatrix} -x_2 \\ x_1 \\ 0 \end{pmatrix}, \quad S = \frac{s(r, x_3)}{r} \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}, \quad T = t(r, x_3) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Advantage of Q: div $Q = 0$. Therefore: symmetric Sobolev-spaces

$$\mathcal{H}^k = \left\{ Q : Q \in H^k(\mathbb{R}^3) \right\}, \quad k \in \mathbb{N}_0$$

On $\mathcal{H}^1$: $\|\nabla U\|_{L^2} = \|\nabla \times U\|_{L^2}$

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Existence results in $\mathcal{H}^1$

$$J[U] = \int_{\mathbb{R}^3} \frac{1}{2}|\nabla \times U|^2 + \frac{V(x)}{2}|U|^2 - \frac{\Gamma(x)}{p+1}|U|^{p+1}\, dx, \quad U \in \mathcal{H}^1$$

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Defocusing case:

Benci-Fortunato (1975): $X \hookrightarrow L^2$. Hence: J has a minimizer $\neq 0$.

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Focusing case:

$$\mathcal{H}^1 = \mathcal{H}^+ \oplus \mathcal{H}^-$$

Least energy level:

$$c = \inf_N J[U], \quad N = \left\{U \in \mathcal{H}^1 \setminus \{0\} : J'[U]\phi = 0 \;\forall \phi \in \mathcal{H}^- \oplus [U]\right\}$$

$= \text{Nehari-Pankov manifold}$

natural constraint, i.e., no Lagrange multiplier

$J|_N$ is bounded from below

$\exists$ bounded sequence $U_k \in N$: $J[U_k] \to c$, $J'[U_k] \to 0$

concentration compactness: $\exists$ min. sequence $\widetilde{U}_k \rightharpoonup U_0 \neq 0$

Example of a potential V with $0 \notin \sigma(L)$

Consider potential $V(r, x_3) = W(r) + P(x_3)$

$$L = \nabla \times \nabla \times + V(r, x_3), \qquad \sigma(L) = \sigma(L_r) + \sigma(L_P)$$

$$\text{where } L_r = -\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right) + \frac{1}{r^2} + W(r), \quad L_P = -\frac{d^2}{dx_3^2} + P(x_3)$$

assume $P =$periodic, $\sigma(L_P) = [\nu_1, \nu_2] \cup [\nu_3, \nu_4] \cup \ldots$

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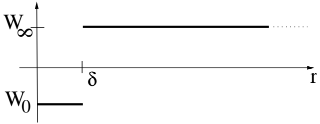
Define

$$W(r) := \begin{cases} W_0, & 0 \le r \le \delta, \\ W_\infty, & \delta \le r < \infty. \end{cases}$$



Take any $\mu_0 < W_\infty$. $\Rightarrow \exists W_0, \delta$ s.t. $\sigma(L_r) = \{\mu_0\} \cup [W_\infty, \infty)$.

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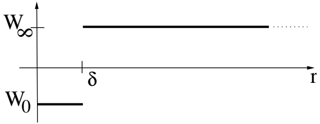

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$$\sigma(L) = \sigma(L_r) + \sigma(L_P) = [\mu_0 + \nu_1, \mu_0 + \nu_2] \cup [\mu_0 + \nu_3, \mu_0 + \nu_4] + \dots$$
$$\cup [\nu_1 + W_\infty, \infty)$$

Sufficient for $0 \notin \sigma(L)$: $\quad -W_\infty < \nu_1 < \nu_2 < -\mu_0 < \nu_3$

Summary

Curl-curl problem:

$$(\nabla \times \nabla \times + V(x))\, U = \Gamma(x)|U|^{p-1} U \text{ in } \mathbb{R}^3, \quad U \in H^1.$$

- Use spaces of cylindrical symmetry: $U(r, x_3) = \frac{q(r,x_3)}{r} \begin{pmatrix} -x_2 \\ x_1 \\ 0 \end{pmatrix}$
- Assume $V = V(r, x_3)$, $\Gamma = \Gamma(r, x_3)$
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